Solution to STAT 517 HW2

6.5.1

$$\bar{x} = .6, \sum (x_i - \bar{x})^2 = 3.6, n = 10$$

$$H_0: \theta_1 = 0, H_a: \theta_1 \neq 0$$

$$\text{Reject when } |T| = \frac{\sqrt{n}\,(\bar{x})}{\sqrt{\sum (x_i - \bar{x})^2/(n-1)}} > t_{0.025,9} = 2.262$$

So we reject H0 after we plug in everything.

6.5.4 $\lambda = \dfrac{\left\{\frac{1}{(2\pi)[(\sum x_i^2 + \sum y_i^2)/(n+m)]}\right\}^{(n+m)/2}}{\left[\frac{1}{(2\pi)(\sum x_i^2/n)}\right]^{n/2}\left[\frac{1}{(2\pi)(\sum y_i^2/m)}\right]^{m/2}} \le k,$

which is equivalent to $F \le c_1$ or $F \ge c_2$, where $F = \frac{\sum X_i^2/n}{\sum Y_i^2/m}$ has an $F(r_1 = n, r_2 = m)$ distribution when $\theta_1 = \theta_2$.

6.5.6 Note $\hat{\theta}_i = \max\{-1\text{st order statistic}, n\text{th order statistic}\}$, where $n = n_1 = n_2$. Hence, in a notation that seems clear, we have

$$\lambda = \frac{1/[2\max(\hat{\theta}_X, \hat{\theta}_Y)]^{2n}}{[1/(2\hat{\theta}_X)^n][1/(2\hat{\theta}_Y)^n]} = \left[\frac{\min(\hat{\theta}_X, \hat{\theta}_Y)}{\max(\hat{\theta}_X, \hat{\theta}_Y)}\right]^n.$$

If $U = \min(\hat{\theta}_X, \hat{\theta}_Y)$and $V = \max(\hat{\theta}_X, \hat{\theta}_Y)$, the joint pdf is

$$g(u, v) = 2n^2 u^{n-1} v^{n-1}/\theta^{2n}, \quad 0 < u < v < \theta.$$

So the distribution function of λ is

$$\begin{aligned} H(z) &= P(U \le z^{1/n} V), \quad 0 \le z \le 1, \\ &= \int_0^\theta \int_0^{z^{1/n}v} g(u, v)\, du\, dv \\ &= \int_0^\theta 2nzv^{2n-1}/\theta^{2n}\, dv \\ &= z, \quad 0 \le z \le 1, \end{aligned}$$

which is uniform (0, 1). Thus $-2\log\lambda$ is $\chi^2(2)$, where the degrees of freedom $= 2 = 2$(dimension of Ω − dimension of ω). Note that this is a nonregular case.

6.5.11 Under H_0, $p_1 = p_2 = p$. Thus both $\overline{X}$ and $\overline{Y}$ are consistent estimators of p. Hence

$$\begin{aligned} \hat{p} &= \frac{n_1}{n}\overline{X} + \frac{n_2}{n}\overline{Y} \\ &\xrightarrow{P} \lambda_1 p + \lambda_2 p = p. \end{aligned}$$

7.1.1 E$\bar{x}$=θ and var($\bar{x}$) = θ^2/n

7.1.4 K2=2/3 and K1=1/3 can minimize $3k_2^2 - 4k_2 + 2$, which can minimize the variance.

7.1.6 We have that $E(Y) = n\theta$, $\text{var}(Y) = n\theta$. Thus

$$E[(\theta - b - Y/n)^2] = (y - b - \theta)^2 + (n\theta)(1/n)^2 = b^2 + \theta^2/n.$$

Thus take $b = 0$ and us $\delta(y) = y/n$. Clearly $\max(\theta^2/n)$ does not exist.