STAT 517 HW9 Solution

8.5.2 (a) and (b) are straightforward.

(c) (1) $P(\sum X_i \geq c; \theta = 1/2) = (2)P(\sum X_i < c; \theta = 1)$ where $\sum X_i$ is Poisson (10θ). Using the Poisson tables, we find, with $c = 6$, the left side is too large, namely $1 - 0.616 > (2)(0.067)$. With $c = 7$, the left side is too small, namely $1 - 0.762 < 2(0.130)$ or, equivalently, $0.238 < 0.260$. To make this last inequality an equality, we need part of the probability that $Y = 6$, namely 0.146 and 0.063 under the respective hypotheses. So $0.238 + 0.146p = 0.260 - 2(0.063)p$ and $p = 0.08$.

8.5.5

$$\frac{\frac{1}{(1)(5)}\exp\left(-\frac{x}{1} - \frac{y}{5}\right)}{\frac{1}{(3)(2)}\exp\left(-\frac{x}{3} - \frac{y}{2}\right)} = \frac{6}{5}\exp\left(-\frac{2x}{3} + \frac{3y}{10}\right) \leq k$$
$$-\frac{2x}{3} + \frac{3y}{10} \leq \log\frac{5k}{6} = c$$

leads to classification as to (x, y) coming from the second distribution.

9.1.3 Let the two samples be X_1 and $(X_2, \ldots, X_{n-1})$. Then, since $(X_1 - X_1)^2 = 0$,

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=2}^{n}(X_i - \bar{X}') + [(X_1 - \bar{X})^2 + (n-1)(\bar{X}' - \bar{X})^2].$$

If we write $\bar{X} = [X_1 + (n-1)\bar{X}']/n$, it is easy to show that the second term on the right side is equal to $(n-1)(X_1 - \bar{X}')^2/n$, and it is $\chi^2(1)$ after being divided by σ^2.

9.2.1 It is easy to show the first equality by writing

$$\sum_{j=1}^{b}\sum_{i=1}^{a_j}(X_{ij} - \bar{X}_{..})^2 = \sum\sum[(X_{ij} - \bar{X}_{.j}) + (\bar{X}_{.j} - \bar{X}_{..})]^2$$

and squaring the binomial on the right side (the sum of the cross product term cleary equals zero).

9.2.3

$$Cov(X_{ij} - \bar{X}_{.j}, X_{.j} - \bar{X}_{..}) = 0$$

So they are uncorrelated.

9.2.7

$$F = \frac{Q_4/(b-1)}{Q_3/(a-1)b} = 10.22 > critical\ value\ 4.26$$

We reject H0: u1 = u2 = u3