STAT 517 HW7 SOLUTION:

8.1.2

{x1, x2 : x1 + x2 >= c} is the best critical region.

8.1.5

$$\frac{1^n}{(2x_1)(2x_2)\dots(2x_n)} \le k \text{ implies that } c = \frac{1}{2^n k} \le \prod_{i=1}^{n} x_i.$$

8.1.10

$$\frac{\frac{(0.1)^{\sum x_i} e^{-n(0.1)}}{x_1!x_2!\cdots x_n!}}{\frac{(0.5)^{\sum x_i} e^{-n(0.5)}}{x_1!x_2!\cdots x_n!}} \le k; \quad \frac{e^{n(0.4)}}{k} \le 5^{\sum x_i}; \quad c \le \sum_{i=1}^{n} x_i.$$

If $n = 10$ and $c = 3$; $\gamma(\theta) = P_\theta(\sum X_i \ge 3)$. So $\alpha = \gamma(0.1) = 0.08$ and $\gamma(0.5) = 0.875$.

8.2.3

$$\begin{aligned}\gamma(\theta) &= P_\theta(\bar{X} \ge 3/5) = P_\theta\left(\frac{\bar{X}-\theta}{2/5} \ge \frac{3/5-\theta}{2/5}\right)\\ &= 1 - \Phi\left(\frac{3-5\theta}{5}\right).\end{aligned}$$

8.2.6 If $\theta > \theta'$, then we want to use a critical region of the from $\sum x_i^2 \ge c$. If $\theta < \theta'$, the critical region is like $\sum x_i^2 \le c$. That is, we cannot find one test which will be best for each type of alternative.

8.2.9 Let $X_1, X_2, \ldots, X_n$ be a random sample with the common Bernoulli pmf with parameter as given in the problem. Based on Example 8.2.5, the UMP test rejects H_0 if $Y \geq c$, $Y = \sum_{i=1}^{n} X_i$. In general, Y has a binomial(n, θ) distribution. To determine n we solve two simultaneous equations, one involving level and the other power. The level equation is

$$\begin{aligned} 0.05 &= \gamma(1/20) = P_{1/20}\left[\frac{Y - (n/20)}{\sqrt{19n/400}} \geq \frac{c - (n/20)}{\sqrt{19n/400}}\right] \\ &\doteq P\left[Z \geq \frac{c - (n/20)}{\sqrt{19n/400}}\right], \end{aligned}$$

where by the Central Limit Theorem Z has a standard normal distribution. Hence, we get the equation

$$\frac{c - (n/20)}{\sqrt{19n/400}} = 1.645. \tag{8.0.1}$$

Likewise from the desired power $\gamma(1/10) = 0.90$, we obtain the equation

$$\frac{c - (n/20) - (n/10)}{\sqrt{9n/100}} = -1.282. \tag{8.0.2}$$

Solving (8.0.1) and (8.0.2) simultaneously, gives the solution $n = 122$.