STAT 517 MIDTERM: 40 pt total

1. (10pt) You have a a sample of i.i.d random variables with distribution function $F(t) = t^{\alpha}$ when $t \in (0, 1)$, $F(t) = 0$ for $t \leq 0$ and $F(t) = 1$ for $t \geq 1$. Find an MLE of $\alpha > 0$ using the maximum likelihood method

2. (10pt)A car company introduces a new car model and advertises it as more economical than other cars of the same class. The specific claim is that the new car will run 15 kilometers per liter on the highway against 12 for competitors. Assume that you have a sample of $n = 5$ cars and that the mileage $Y_i \sim N(\mu, \sigma^2)$ with $\sigma^2 = 4$. You must test $H_0 : \mu = 12$ vs. $H_a : \mu = 15$. If you adopt the rejection region $\bar{Y} \geq 14$

   (a) (3pt)What is the significance level of the test?

   (b) (3pt)What is the power of the test?

   (c) (4pt)If you adopt $\alpha = 0.05$ what is the critical value C such that the rejection region must be $\bar{Y} \geq C$? Express your solution using normal distribution percentiles.

3. (10pt) A student takes a bus from his home to the University campus. Over a period of 30 days he records bus waiting times. Assume that waiting times $t_i \sim Exp(\theta)$; that is, the pdf is $f_\theta(t) = \theta e^{-\theta t}$ for $t \geq 0$. Also, the average waiting time over these 8 days is $\bar{t} = 8$ min. Find a 95% confidence interval for the average waiting time. Hint: for $X \sim Exp(\theta)$ $\theta X \sim Exp(1) = \frac{1}{2}\chi^2_2$

4. (10pt) Let T_{1n} and T_{2n} be two unbiased estimators of the parameter θ

   (a) (2pt)Show that for any $0 < \alpha < 1$ $T_n = \alpha T_{1n} + (1 - \alpha) T_{2n}$ is also unbiased

   (b) (2pt)Now also assume that T_{1n} and T_{2n} are uncorrelated. If $VarT_{1n} = \sigma_{1n}^2$ and $VarT_{2n} = \sigma_{2n}^2$, calculate $VarT_n$

   (c) (2pt)Find the value of $\alpha \in [0, 1]$ such that $VarT_n$ is minimized

   (d) (4pt)Show that T_n is consistent when $\sigma_{1n}^2 \to 0$ and $\sigma_{2n}^2 \to 0$ as $n \to \infty$