STAT 517 FINAL; 40pt total

1. (8pt) Let $X \sim b(1, p)$ be a Bernoulli random variable. Show that it is impossible to obtain an unbiased estimator of p^2 .

- 2. (10pt) Let X_1, \ldots, X_n be independent random variables with discrete uniform distribution; that is, $p_N(k) = P(X_i = k) = \frac{1}{N}$ for any $k = 1, \ldots, N$ where N > 0 is a parameter.
 - (4pt)Show that $Y_n = \max_{1 \le i \le n} X_i$ is a sufficient statistic for N
 - (3pt)Show that the family of pmf's $p_N(k)$ is complete
 - (3pt) Show that the family of pmf's of Y_n is also complete and, therefore, Y_n is a complete sufficient statistic for N

- 3. (10pt)Let $X \sim P(\lambda)$ be a single Poisson observation; thus, the sample size is n = 1. Our task is to estimate $P(X = 0) = e^{-\lambda}$
 - (5pt) Suggest a simple unbiased estimator of $e^{-\lambda}$ and find its Cramer-Rao lower bound
 - (5pt)Show that there can be only one unbiased estimator of $e^{-\lambda}$ and, therefore, the estimator you proposed is an UMVUE.

4. (12pt)Let $X \sim b(1, p)$ where $p \in \left[\frac{1}{4}, \frac{3}{4}\right]$.

- (4pt)Find the maximum likelihood estimator of p
- (4pt)Is this MLE unbiased?
- (4pt)Find the variance of this MLE and its mean squared error (recall, the mean squared error is the sum of variance and squared bias). Is this MLE better or worse than the trivial estimator δ(X) = ¹/₂ in terms of its MSE?