37. The joint pmf of X_1 and X_2 is presented below. Each joint probability is calculated using the independence of X_1 and X_2; e.g., $p(25, 25) = P(X_1 = 25) \cdot P(X_2 = 25) = (.2)(.2) = .04$.

	$p(x_1, x_2)$	x_1: 25	40	65	
x_2	25	.04	.10	.06	.2
	40	.10	.25	.15	.5
	65	.06	.15	.09	.3
		.2	.5	.3	

a. For each coordinate in the table above, calculate $\bar{x}$. The six possible resulting $\bar{x}$ values and their corresponding probabilities appear in the accompanying pmf table.

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	.04	.20	.25	.12	.30	.09

From the table, $E(\bar{X}) = (25)(.04) + 32.5(.20) + ... + 65(.09) = 44.5$. From the original pmf, $\mu = 25(.2) + 40(.5) + 65(.3) = 44.5$. So, $E(\bar{X}) = \mu$.

b. For each coordinate in the joint pmf table above, calculate $s^2 = \frac{1}{2-1}\sum_{i=1}^{2}(x_i - \bar{x})^2$. The four possible resulting s^2 values and their corresponding probabilities appear in the accompanying pmf table.

s^2	0	112.5	312.5	800
$p(s^2)$	.38	.20	.30	.12

From the table, $E(S^2) = 0(.38) + \ldots + 800(.12) = 212.25$. From the original pmf, $\sigma^2 = (25 - 44.5)^2(.2) + (40 - 44.5)^2(.5) + (65 - 44.5)^2(.3) = 212.25$. So, $E(S^2) = \sigma^2$.

48.

a. No, it doesn't seem plausible that waist size distribution is approximately normal. The normal distribution is symmetric; however, for this data the mean is 86.3 cm and the median is 81.3 cm (these should be nearly equal). Likewise, for a symmetric distribution the lower and upper quartiles should be equidistant from the mean (or median); that isn't the case here.

If anything, since the upper percentiles stretch much farther than the lower percentiles do from the median, we might suspect a right-skewed distribution, such as the exponential distribution (or gamma or Weibull or …) is appropriate.

b. Irrespective of the population distribution's shape, the Central Limit Theorem tells us that $\bar{X}$ is (approximately) normal, with a mean equal to $\mu = 85$ cm and a standard deviation equal to $\sigma / \sqrt{n} = 15 / \sqrt{277} = .9$ cm. Thus,

$$P(\bar{X} \geq 86.3) = P\left(Z \geq \frac{86.3 - 85}{.9}\right) = 1 - \Phi(1.44) = .0749$$

c. Replace 85 with 82 in (b):

$$P(\bar{X} \geq 86.3) = P\left(Z \geq \frac{86.3 - 82}{.9}\right) = 1 - \Phi(4.77) \approx 1 - 1 = 0$$

That is, if the population mean waist size is 82 cm, there would be almost no chance of observing a sample mean waist size of 86.3 cm (or higher) in a random sample if 277 men. Since a sample mean of 86.3 was actually observed, it seems incredibly implausible that μ would equal 82 cm.

49.

a. 11 P.M. – 6:50 P.M. = 250 minutes. With $T_o = X_1 + \ldots + X_{40}$ = total grading time, $\mu_{T_o} = n\mu = (40)(6) = 240$ and $\sigma_{T_o} = \sigma \cdot \sqrt{n} = 37.95$, so $P(T_o \le 250) \approx$ $P\left(Z \le \frac{250-240}{37.95}\right) = P(Z \le .26) = .6026.$

b. The sports report begins 260 minutes after he begins grading papers. $P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > .53) = .2981.$

58.

a. $E(27X_1 + 125X_2 + 512X_3) = 27E(X_1) + 125E(X_2) + 512E(X_3)$
$= 27(200) + 125(250) + 512(100) = 87{,}850.$
$V(27X_1 + 125X_2 + 512X_3) = 27^2\, V(X_1) + 125^2\, V(X_2) + 512^2\, V(X_3)$
$= 27^2\,(10)^2 + 125^2\,(12)^2 + 512^2\,(8)^2 = 19{,}100{,}116.$

b. The expected value is still correct, but the variance is not because the covariances now also contribute to the variance.

59.

a. $E(X_1 + X_2 + X_3) = 180$, $V(X_1 + X_2 + X_3) = 45$, $SD(X_1 + X_2 + X_3) = \sqrt{45} = 6.708.$
$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{200-180}{6.708}\right) = P(Z \le 2.98) = .9986.$
$P(150 \le X_1 + X_2 + X_3 \le 200) = P(-4.47 \le Z \le 2.98) \approx .9986.$

b. $\mu_{\bar{X}} = \mu = 60$ and $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{\sqrt{15}}{\sqrt{3}} = 2.236$, so
$P(\bar{X} \ge 55) = P\left(Z \ge \frac{55-60}{2.236}\right) = P(Z \ge -2.236) = .9875$ and
$P(58 \le \bar{X} \le 62) = P(-.89 \le Z \le .89) = .6266.$

c. $E(X_1 - .5X_2 - .5X_3) = \mu - .5\,\mu - .5\,\mu = 0$, while
$V(X_1 - .5X_2 - .5X_3) = \sigma_1^2 + .25\sigma_2^2 + .25\sigma_3^2 = 22.5 \Rightarrow SD(X_1 - .5X_2 - .5X_3) = 4.7434$. Thus,
$P(-10 \le X_1 - .5X_2 - .5X_3 \le 5) = P\left(\frac{-10-0}{4.7434} \le Z \le \frac{5-0}{4.7434}\right) = P(-2.11 \le Z \le 1.05) = .8531 - .0174 =$ $.8357.$

d. $E(X_1 + X_2 + X_3) = 150$, $V(X_1 + X_2 + X_3) = 36 \Rightarrow SD(X_1 + X_2 + X_3) = 6$, so
$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{160-150}{6}\right) = P(Z \le 1.67) = .9525.$
Next, we want $P(X_1 + X_2 \ge 2X_3)$, or, written another way, $P(X_1 + X_2 - 2X_3 \ge 0)$.
$E(X_1 + X_2 - 2X_3) = 40 + 50 - 2(60) = -30$ and $V(X_1 + X_2 - 2X_3) = \sigma_1^2 + \sigma_2^2 + 4\sigma_3^2 = 78 \Rightarrow$
$SD(X_1 + X_2 - 2X_3) = 8.832$, so
$P(X_1 + X_2 - 2X_3 \ge 0) = P\left(Z \ge \frac{0-(-30)}{8.832}\right) = P(Z \ge 3.40) = .0003.$