1. Suppose that in a certain population 30% of couples have one child, 50% have two children, and 20% have three children. One family is picked at random from this population. What is the expected number of boys in the family? Assume that the probability of a childbirth resulting in a boy is 0.5.

2. Let the random variables $X$ and $Y$ have the joint pmf

   (a) $p(x, y) = \frac{1}{3}$ for $(x, y) = (0, 0), (1, 1), (2, 2)$ and zero elsewhere

   (b) $p(x, y) = \frac{1}{3}$ for $(x, y) = (0, 2), (1, 1), (2, 0)$ and zero elsewhere

   Find the correlation $\rho_{X,Y}$ in both cases.

3. Let $U$ and $V$ be two random variables with common mean and common variance. Let $X = U + V$ and $Y = U - V$. Show that $X$ and $Y$ are uncorrelated.

4. Consider the trinomial distribution with pmf

$$P(X = x, Y = y) = \frac{n!}{x!y!(n - x - y)!} p_1^x p_2^y p_3^{n-x-y}$$

   where $x, y$ are non-negative integers such that $x + y \leq n$. Of course, $p_1, p_2, p_3 \geq 0$ and $p_1 + p_2 + p_3 = 1$.

   (a) Find the conditional distribution $P(Y = y|X = x)$

   (b) What is $E(Y|X = x)$? Can you find $E(X|Y = y)$ without any additional computations?