1. Suppose the distribution of heights in a population is approximately normal. 10% of individuals are over 6 feet tall, and the average height is 5ft.6in. What is the probability that a randomly selected individual from this population is over 6ft. 1in. tall? Keep in mind that 1 ft is equal to 12 inches.

   Solution: height is $X \sim N(\mu, \sigma^2)$ where $\mu = 66$in. Since 6ft is the same as 72 in., we have the probability $P(X > 72) = 0.1$ which implies that 72 is the 90%th percentile of X. Therefore, we have $72 = 66 + 1.282\sigma$ where 1.282 is the 90%th percentile of $N(0, 1)$. From this we find that $\sigma = 4.68$ Therefore, the probability that one individual is taller than 6ft. 1in. is $P(X > 73) = P(Z > (73 - 66)/4.68) = P(Z > 1.496) = 0.0674$.

2. Suppose $X \sim N(0, 1)$, $Y \sim N(0, 9)$, and X and Y are independent. Find the mean, the variance and the third moment of $X + Y$

   Solution: $X + Y \sim N(0, 10)$. Therefore, its mean is equal to zero, its variance is 10, its third moment is zero, and its fourth moment is $\frac{1}{\sqrt{10}} \times 3 = \frac{3}{\sqrt{10}}$.

3. Normal approximation to binomial probabilities is routinely used in designing polls on an issue, for example polls to predict a winner in an election. Suppose that in an election there are two candidates, A and B, and among all voters, 52% support A and 48% support B. A poll of 1400 voters is done. What is the probability that the poll will predict a correct winner?

   Solution: let X be the number of respondents in the poll who favor A. The poll will predict the correct winner if $X > 700$. By using the continuity corrected normal approximation,

$$P(X > 700) = 1 - P(X \leq 700) \approx 1 - \Phi\left(\frac{700.56 - 1400 \times .52}{\sqrt{1400 \times .52 \times .48}}\right)$$
$$= 1 - \Phi(-1.5) = \Phi(1.5) = 0.9332$$