Homework 3

1. A 14 inch circular pizza has been baked with 20 pieces of barbecued chicken. At a party, you were served with a $4 \times 4 \times 2$ (in inches) triangular slice. What is the probability that you got at least one piece of chicken? Assume Poisson distribution for a number of chicken pieces per square inch.

   Solution: the area of the triangle as given should be $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$. In our case, it is equal to $\sqrt{15} \approx 3.87$. The area of a circle of radius 7 is $\pi * 7^2 = 153.94$. Thus, we model the number of pieces of barbecued chicken per a triangular slice as $X \sim P(\lambda)$ with $\lambda = 20 * 3.87/153.94 = .503$. With this in mind, $P(X \geq 1) = 1 - e^{-.503} = .395$

2. Suppose $X \sim Unif[0,1]$. Let event $C = \{\sin\left(\frac{\pi}{2}X\right) \geq \frac{1}{\sqrt{2}}\}$ and $D = \{X \text{ is a rational number }\}$. Find $P(C)$ and $P(D)$.

   Solution: for the event C, $\sin\left(\frac{\pi}{2}X\right) \geq \frac{1}{\sqrt{2}}$ if and only if $\frac{\pi}{2}X \geq \frac{\pi}{4}$ which implies that $X \geq \frac{1}{2}$. Thus,

   $$P(C) = P\left(X \geq \frac{1}{2}\right) = \int_{1/2}^{1} dx = \frac{1}{2}$$

   Note that $P(D) = \sum_{x:\ x \text{ is rational}} P(X = x) = \sum_{x:\ x \text{ is rational}} 0 = 0$

3. Suppose $X \sim Unif[0,1]$. Find the distribution of $Y = X^k$ for any general positive integer $k = 2, 3, 4, \ldots$ (not just $Y = X^2$ as we did in class).

   Solution: using the Jacobian transformation, we find that

   $$f_Y(y) = \frac{f(g^{-1}(y))}{|g'(g^{-1}(y))|} = \frac{f(y^{1/k})}{ky^{k-1/k}} = \frac{1}{ky^{k-1/k}}$$

   for any $0 \leq y \leq 1$.

4. Suppose $X \sim Unif[0,1]$. Find $E(\log X)$

   Solution: $E(\log X) = \int_0^1 \log x \, dx = \int_{-\infty}^0 ye^y \, dy = -1$.