Homework 3

1. Suppose X us the sum of two rolls of a fair die. Then, we can write $X = X_1 + X_2$ where X_1 and X_2 are the numbers obtained on the two rolls, respectively. Use our result for the MGF of the discrete uniform distribution with $n = 6$ to obtain the MGF of X.

   Solution: by direct multiplication, we have

$$\psi_X(t) = \left[\frac{e^t(e^{6t}-1)}{6(e^t-1)}\right]^2 = \frac{e^{2t}(e^{6t}-1)^2}{36(e^t-1)^2}$$

2. Ms. Smith drives into town once a week to by groceries. In the past, she parked her car at a lot for five dollars but she decided that, for the next five weeks, she will park at the fire hydrant and risk getting tickets with fines of 25 dollars per offense. If the probability of getting a ticket is 0.1, what is the probability that she will pay more in fines in five weeks than she would pay in parking fees if she had opted not to park by the fire hydrant?

   Solution: let X be the number of weeks among the next five weeks in which she gets a ticket. Clearly, $X \sim Bin(5, 0.1)$. Ms. Smith's parking fees would have been 25 dollars for the five weeks combined if she did not park by the hydrant. Thus, the required probability is

$$P(25X > 25) = P(X > 1) = 1 - [P(X = 0) + P(X = 1)] = 1$$
$$- \left[(0.9)^5 + \binom{5}{1}(0.1)(0.9)^4\right] = 0.0815$$

3. Suppose a door-to-door salesman makes an actual sale in 25% of the visits he makes. He is supposed to make at least two sales per day. How many visits should plan on making to be 90% sure of making at least two sales?

   Solution: let X be the visit at which the second sale is made. Then, $X \sim NB(r, p)$ with $r = 2$ and $p = 0.25$ Therefore, X has the pmf $P(X = x) = (x-1)(0.25)^2(.75)^{x-2}$ where $x = 2, 3, \ldots$ Summing, for any given k, $P(X > k) = \sum_{x=k+1}^{\infty}(x-1)(0.25)^2(.75)^{x-2} = \frac{k+3}{3}(3/4)^k$. We want $\frac{k+3}{3}(3/4)^k \leq 0.1$ By computing this directly, we find that $P(X > 15) < 0.1$ but $P(X > 14) > 0.1$ Thus, the salesman should plan on making 15 visits.