1. One of two urns has a red and b black balls, and the other has c red and d black balls. One ball is chosen at random from each urn, and then one of these two balls is chosen at random. What is the probability that this ball is red? Assume that a=99, b=1, c=1, and d=1. What is the numeric answer? Does it surprise you?

Solution: P(final ball is red)=a/(a+b)\*c/(c+d)+1/2\*[a/(a+b)\*d/(c+d)+b/(a+b)\*c/(c+d)]=

=(2ac+ad+bc)/(2(a+b)(c+d)). For given number it is about 75%.

1. Suppose a fair die is rolled twice, and let A and B be the events that the sum of two rolls is 7 and the first roll j, where j is any given number 1,2,…, 6. Are A and B independent events?

Solution: P(A)=6/36=1/6, P(B)=1/6, and P$\left(A∩B\right)=1/36$. Thus, A and B are independent events.

1. Peter and Karen take turns, starting with Karen, rolling a fair die. The ﬁrst to obtain a six wins. What is the probability that the winner is Karen?

Solution: P(Karen wins)=1/6+5/6\*5/6\*1/6+5/6\*5/6\*5/6\*1/6+…=1/6$\left(1+\sum\_{i=1}^{\infty }\left(\frac{5}{6}\right)^{2i}\right)$=1/6\*$\left(1+\frac{25/36}{1-25/36}\right)=6/11$

1. Suppose that the questions in a multiple choice exam have five alternatives each, of which a student picks one as the correct alternative. A student either knows the truly correct alternative with probability 0.7 or he/she randomly picks one of the five alternatives as his choice. Suppose a particular problem was answered correctly. What is the probability that the student really knew the correct answer?

Solution: Let A=the student knew the correct answer and B=the student answered the question correctly. We want to compute P(A|B). By Bayes’ theorem,

P(A|B)=$\frac{P(B|A)P(A)}{P\left(A\right)P\left(A\right)+P\left(A^{'}\right)P(B|A^{)'}}=\frac{1\*0.7}{1\*0.7+0.2\*0.3}=0.921$