STAT 511 HW7 solutions

8.2

15. In each case, the direction of H_a indicates that the P-value is $P(Z \geq z) = 1 - \Phi(z)$.

a. P-value $= 1 - \Phi(1.42) = .0778$.

b. P-value $= 1 - \Phi(0.90) = .1841$.

c. P-value $= 1 - \Phi(1.96) = .0250$.

d. P-value $= 1 - \Phi(2.48) = .0066$.

e. P-value $= 1 - \Phi(-.11) = .5438$.

17.

a. $z = \dfrac{30{,}960 - 30{,}000}{1500/\sqrt{16}} = 2.56$, so P-value $= P(Z \geq 2.56) = 1 - \Phi(2.56) = .0052$.
Since $.0052 < \alpha = .01$, reject H_0.

b. $z_\alpha = z_{.01} = 2.33$, so $\beta(30500) = \Phi\left(2.33 + \dfrac{30000 - 30500}{1500/\sqrt{16}}\right) = \Phi(1.00) = .8413$.

c. $z_\alpha = z_{.01} = 2.33$ and $z_\beta = z_{.05} = 1.645$. Hence, $n = \left[\dfrac{1500(2.33 + 1.645)}{30{,}000 - 30{,}500}\right]^2 = 142.2$, so use $n = 143$.

d. From (a), the P-value is .0052. Hence, the smallest α at which H_0 can be rejected is .0052.

24. Let μ denote the true average estimated calorie content of this 153-calorie beer. The hypotheses of interest are H_0: $\mu = 153$ v. H_a: $\mu > 153$. Using z-based inference with the data provided, the P-value of the test is $P\left(Z \geq \dfrac{191 - 153}{89/\sqrt{58}}\right) = 1 - \Phi(3.25) = .0006$. At any reasonable significance level, we reject the null hypothesis. Therefore, yes, there is evidence that the true average estimated calorie content of this beer exceeds the actual calorie content.

8.3

29. The hypotheses are H_0: $\mu = .5$ versus H_a: $\mu \neq .5$. Since this is a two-sided test, we must double the one-tail area in each case to determine the P-value.

a. $n = 13 \Rightarrow \text{df} = 13 - 1 = 12$. Looking at column 12 of Table A.8, the area to the right of $t = 1.6$ is .068. Doubling this area gives the two-tailed P-value of $2(.068) = .134$. Since $.134 > \alpha = .05$, we do not reject H_0.

b. For a two-sided test, observing $t = -1.6$ is equivalent to observing $t = 1.6$. So, again the P-value is $2(.068) = .134$, and again we do not reject H_0 at $\alpha = .05$.

c. $\text{df} = n - 1 = 24$; the area to the left of -2.6 = the area to the right of $2.6 = .008$ according to Table A.8. Hence, the two-tailed P-value is $2(.008) = .016$. Since $.016 > .01$, we do not reject H_0 in this case.

d. Similar to part (c), Table A.8 gives a one-tail area of .000 for $t = \pm 3.9$ at $\text{df} = 24$. Hence, the two-tailed P-value is $2(.000) = .000$, and we reject H_0 at any reasonable α level.

31. This is an upper-tailed test, so the P-value in each case is $P(T \geq \text{observed } t)$.

a. P-value = $P(T \geq 3.2 \text{ with df} = 14) = .003$ according to Table A.8. Since $.003 \leq .05$, we reject H_0.

b. P-value = $P(T \geq 1.8 \text{ with df} = 8) = .055$. Since $.055 > .01$, do not reject H_0.

c. P-value = $P(T \geq -.2 \text{ with df} = 23) = 1 - P(T \geq .2 \text{ with df} = 23)$ by symmetry $= 1 - .422 = .578$. Since .578 is quite large, we would not reject H_0 at any reasonable α level. (Note that the sign of the observed t statistic contradicts H_a, so we know immediately not to reject H_0.)

8.4

43.

a. The parameter of interest is p = the proportion of the population of female workers that have BMIs of at least 30 (and, hence, are obese). The hypotheses are H_0: $p = .20$ versus H_a: $p > .20$.
With $n = 541$, $np_0 = 541(.2) = 108.2 \geq 10$ and $n(1 - p_0) = 541(.8) = 432.8 \geq 10$, so the "large-sample" z procedure is applicable.
From the data provided, $\hat{p} = \dfrac{120}{541} = .2218$, so $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \dfrac{.2218 - .20}{\sqrt{.20(.80)/541}} = 1.27$ and P-value $= P(Z \geq 1.27) = 1 - \Phi(1.27) = .1020$. Since $.1020 > .05$, we fail to reject H_0 at the $\alpha = .05$ level. We do not have sufficient evidence to conclude that more than 20% of the population of female workers is obese.

b. A Type I error would be to incorrectly conclude that more than 20% of the population of female workers is obese, when the true percentage is 20%. A Type II error would be to fail to recognize that more than 20% of the population of female workers is obese when that's actually true.

c. The question is asking for the chance of committing a Type II error when the true value of p is .25, i.e. $\beta(.25)$. Using the textbook formula,

$$\beta(.25) = \Phi\left[\frac{.20 - .25 + 1.645\sqrt{.20(.80)/541}}{\sqrt{.25(.75)/541}}\right] = \Phi(-1.166) \approx .121.$$