44.

a. Let p = true proportion of all nickel plates that blister under the given circumstances. The hypotheses are H_0: $p = .10$ versus H_a: $p > .10$. Using the one-proportion z procedure, the test statistic is $z = \dfrac{14/100 - .10}{\sqrt{.10(.90)/100}} = 1.33$ and the P-value is $P(Z \geq 1.33) = 1 - \Phi(1.33) = .0918$. Since $.0918 > .05$, we fail to Reject H_0. The data does not give compelling evidence for concluding that more than 10% of all plates blister under the circumstances.

The possible error we could have made is a Type II error: failing to reject the null hypothesis when it is actually true.

b. $\beta(.15) = \Phi\left[\dfrac{.10 - .15 + 1.645\sqrt{.10(.90)/100}}{\sqrt{.15(.85)/100}}\right] = \Phi(-.02) = .4920$. When $n = 200$,

$$\beta(.15) = \Phi\left[\frac{.10 - .15 + 1.645\sqrt{.10(.90)/200}}{\sqrt{.15(.85)/200}}\right] = \Phi(-.60) = .2743$$

c. $n = \left[\dfrac{1.645\sqrt{.10(.90)} + 1.28\sqrt{.15(.85)}}{.15 - .10}\right]^2 = 19.01^2 = 361.4$, so use $n = 362$.

46.

a. Let X = the number of couples who lean more to the right when they kiss. If $n = 124$ and $p = 2/3$, then $E[X] = 124(2/3) = 82.667$. The researchers observed $x = 80$, for a difference of 2.667. The probability in question is $P(|X - 82.667| \geq 2.667) = P(X \leq 80 \text{ or } X \geq 85.33) = P(X \leq 80) + [1 - P(X \leq 85)] = B(80;124,2/3) + [1 - B(85;124,2/3)] = 0.634$. (Using a large-sample z-based calculation gives a probability of 0.610.)

b. We wish to test H_0: $p = 2/3$ v. H_a: $p \neq 2/3$. From the data, $\hat{p} = \dfrac{80}{124} = .645$, so our test statistic is $z = \dfrac{.645 - .667}{\sqrt{.667(.333)/124}} = -0.51$. We would fail to reject H_0 even at the $\alpha = .10$ level, since the two-tailed P-value is quite large. There is no statistically significant evidence to suggest the $p = 2/3$ figure is implausible for right-leaning kissing behavior.

50. Notice that with the relatively small sample size, we should use a binomial model here.

a. The alternative of interest here is H_a: $p > .50$ (which states that more than 50% of all enthusiasts prefer gut). So, we'll reject H_0 in favor of H_a when the observed value of X is quite large (much more than 10). Suppose we reject H_0 when $X \geq x$; then $\alpha = P(X \geq x$ when H_0 is true$) = 1 - B(x - 1; 20, .5)$, since $X \sim \text{Bin}(20, .5)$ when H_0 is true.

By trial and error, $\alpha = .058$ if $x = 14$ and $\alpha = .021$ if $x = 15$. Therefore, a significance level of exactly $\alpha = .05$ is not possible, and the largest possible value less than .05 is $\alpha = .021$ (occurring when we elect to reject H_0 iff $X \geq 15$).

b. $\beta(.6) = P(\text{do not reject } H_0 \text{ when } p = .6) = P(X < 15 \text{ when } X \sim \text{Bin}(20,.6)) = B(14; 20, .6) = .874$. Similarly, $\beta(.8) = B(14; 20, .8) = .196$.

c. No. Since 13 is not ≥ 15, we would not reject H_0 at the $\alpha = .021$ level. Equivalently, the P-value for that observed count is $P(X \geq 13 \text{ when } p = .5) = 1 - P(X \leq 12 \text{ when } X \sim \text{Bin}(20,.5)) = .132$. Since $.132 > .021$, we do not reject H_0 at the .021 level (or at the .05 level, for that matter).

3. Let μ_1 = the population mean pain level under the control condition and μ_2 = the population mean pain level under the treatment condition.

a. The hypotheses of interest are H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 > 0$. With the data provided, the test statistic value is $z = \dfrac{(5.2-3.1)-0}{\sqrt{\dfrac{2.3^2}{43}+\dfrac{2.3^2}{43}}} = 4.23$. The corresponding P-value is $P(Z \geq 4.23) = 1 - \Phi(4.23) \approx 0$.

Hence, we reject H_0 at the $\alpha = .01$ level (in fact, at any reasonable level) and conclude that the average pain experienced under treatment is less than the average pain experienced under control.

b. Now the hypotheses are H_0: $\mu_1 - \mu_2 = 1$ versus H_a: $\mu_1 - \mu_2 > 1$. The test statistic value is $z = \dfrac{(5.2-3.1)-1}{\sqrt{\dfrac{2.3^2}{43}+\dfrac{2.3^2}{43}}} = 2.22$, and the P-value is $P(Z \geq 2.22) = 1 - \Phi(2.22) = .0132$. Thus we would reject H_0 at the $\alpha = .05$ level and conclude that mean pain under control condition exceeds that of treatment condition by more than 1 point. However, we would not reach the same decision at the $\alpha = .01$ level (because $.0132 \leq .05$ but $.0132 > .01$).

7. Let μ_1 denote the true mean course GPA for all courses taught by full-time faculty, and let μ_2 denote the true mean course GPA for all courses taught by part-time faculty. The hypotheses of interest are H_0: $\mu_1 = \mu_2$ versus H_a: $\mu_1 \neq \mu_2$; or, equivalently, H_0: $\mu_1 - \mu_2 = 0$ v. H_a: $\mu_1 - \mu_2 \neq 0$.

The large-sample test statistic is $z = \dfrac{(\bar{x}-\bar{y})-\Delta_0}{\sqrt{\dfrac{s_1^2}{m}+\dfrac{s_2^2}{n}}} = \dfrac{(2.7186-2.8639)-0}{\sqrt{\dfrac{(.63342)^2}{125}+\dfrac{(.49241)^2}{88}}} = -1.88$. The corresponding two-tailed P-value is $P(|Z| \geq |-1.88|) = 2[1 - \Phi(1.88)] = .0602$.
Since the P-value exceeds $\alpha = .01$, we fail to reject H_0. At the .01 significance level, there is insufficient evidence to conclude that the true mean course GPAs differ for these two populations of faculty.

9.

a. Point estimate $\bar{x}-\bar{y} = 19.9 - 13.7 = 6.2$. It appears that there could be a difference.

b. H_0: $\mu_1 - \mu_2 = 0$, H_a: $\mu_1 - \mu_2 \neq 0$, $z = \dfrac{(19.9-13.7)}{\sqrt{\dfrac{39.1^2}{60}+\dfrac{15.8^2}{60}}} = \dfrac{6.2}{5.44} = 1.14$, and the P-value = $2[P(Z > 1.14)]$ = $2(.1271) = .2542$. The P-value is larger than any reasonable α, so we do not reject H_0. There is no statistically significant difference.

c. No. With a normal distribution, we would expect most of the data to be within 2 standard deviations of the mean, and the distribution should be symmetric. Two sd's above the mean is 98.1, but the distribution stops at zero on the left. The distribution is positively skewed.

d. We will calculate a 95% confidence interval for μ, the true average length of stays for patients given the treatment. $19.9 \pm 1.96\dfrac{39.1}{\sqrt{60}} = 19.9 \pm 9.9 = (10.0, 21.8)$.