16. The implicit hypotheses are H_0: $\mu = 30$ and H_a: $\mu \neq 30$ ("whether μ differs from the target value"). So, in each case, the P-value is $2 \cdot P(Z \geq |z|) = 2 \cdot [1 - \Phi(|z|)]$.

a. P-value $= 2 \cdot [1 - \Phi(|2.10|)] = .0358$.

b. P-value $= 2 \cdot [1 - \Phi(|-1.75|)] = .0802$.

c. P-value $= 2 \cdot [1 - \Phi(|-0.55|)] = .5824$.

d. P-value $= 2 \cdot [1 - \Phi(|1.41|)] = .1586$.

e. P-value $= 2 \cdot [1 - \Phi(|-5.3|)] \approx 0$.

19.

a. Since the alternative hypothesis is two-sided, P-value $= 2\cdot\left[1-\Phi\left(\left|\frac{94.32-95}{1.20/\sqrt{16}}\right|\right)\right] = 2 \cdot [1 - \Phi(2.27)] = 2(.0116) = .0232$. Since $.0232 > \alpha = .01$, we do not reject H_0 at the .01 significance level.

b. $z_{\alpha/2} = z_{.005} = 2.58$, so $\beta(94) = \Phi\left(2.58 + \frac{95-94}{1.20/\sqrt{16}}\right) - \Phi\left(-2.58 + \frac{95-94}{1.20/\sqrt{16}}\right) = \Phi(5.91) - \Phi(0.75) = .2266$.

c. $z_\beta = z_{.1} = 1.28$. Hence, $n = \left[\frac{1.20(2.58+1.28)}{95-94}\right]^2 = 21.46$, so use $n = 22$.

21. The hypotheses are H_0: $\mu = 5.5$ v. H_a: $\mu \neq 5.5$.

a. The P-value is $2\cdot\left[1-\Phi\left(\left|\frac{5.25-5.5}{.3/\sqrt{16}}\right|\right)\right] = 2 \cdot [1 - \Phi(3.33)] = .0008$. Since the P-value is smaller than any reasonable significance level (.1, .05, .01, .001), we reject H_0.

b. The chance of detecting that H_0 is false is the complement of the chance of a type II error. With $z_{\alpha/2} = z_{.005} = 2.58$, $1-\beta(5.6) = 1-\left[\Phi\left(2.58+\frac{5.5-5.6}{.3/\sqrt{16}}\right) - \Phi\left(-2.58+\frac{5.5-5.6}{.3/\sqrt{16}}\right)\right] = 1 - \Phi(1.25) + \Phi(3.91) = .1056$.

c. $n = \left[\frac{.3(2.58+2.33)}{5.5-5.6}\right]^2 = 216.97$, so use $n = 217$.

30. The hypotheses are H_0: $\mu = 7.0$ versus H_a: $\mu < 7.0$. In each case, we want the one-tail area to the left of the observed test statistic.

a. $n = 6 \Rightarrow \text{df} = 6 - 1 = 5$. From Table A.8, $P(T \le -2.3 \text{ when } T \sim t_5) = P(T \ge 2.3 \text{ when } T \sim t_5) = .035$. Since $.035 \le .05$, we reject H_0 at the $\alpha = .05$ level.

b. Similarly, P-value $= P(T \ge 3.1 \text{ when } T \sim t_{14}) = .004$. Since $.004 < .01$, reject H_0.

c. Similarly, P-value $= P(T \ge 1.3 \text{ when } T \sim t_{11}) = .110$. Since $.110 \ge .05$, do not reject H_0.

d. Here, P-value $= P(T \le .7 \text{ when } T \sim t_5)$ because it's a lower tailed test, and this is $1 - P(T > .7 \text{ when } T \sim t_5) = 1 - .258 = .742$. Since $.742 > .05$, do not reject H_0. (Note: since the sign of the t-statistic contradicted H_a, we know immediately not to reject H_0.)

e. The observed value of the test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{6.68 - 7.0}{.0820} = -3.90$. From this, similar to parts (a)-(c), P-value $= P(T \ge 3.90 \text{ when } T \sim t_5) = .006$ according to Table A.8. We would reject H_0 for any significance level at or above .006.

35.

a. The hypotheses are H_0: $\mu = 200$ versus H_a: $\mu > 200$. With the data provided, $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{249.7 - 200}{145.1/\sqrt{12}} = 1.2$; at df = 12 – 1 = 11, P-value = .128. Since .128 > .05, H_0 is not rejected at the $\alpha = .05$ level. We have insufficient evidence to conclude that the true average repair time exceeds 200 minutes.

b. With $d = \dfrac{|\mu_0 - \mu|}{\sigma} = \dfrac{|200 - 300|}{150} = 0.67$, df = 11, and $\alpha = .05$, software calculates power $\approx .70$, so $\beta(300) \approx .30$.

38. μ = the true average percentage of organic matter in this type of soil, and the hypotheses are H_0: $\mu = 3$ versus H_a: $\mu \ne 3$. With $n = 30$, and assuming normality, we use the t test:
$t = \dfrac{\bar{x} - 3}{s/\sqrt{n}} = \dfrac{2.481 - 3}{.295} = \dfrac{-.519}{.295} = -1.759$. At df = 30 – 1 = 29, P-value $= 2P(T > 1.759) = 2(.041) = .082$.
At significance level .10, since $.082 \le .10$, we would reject H_0 and conclude that the true average percentage of organic matter in this type of soil is something other than 3. At significance level .05, we would not have rejected H_0.