32.

a. $P(X \le 15) = P\left(Z \le \frac{15-15.0}{1.25}\right) = P(Z \le 0) = \Phi(0.00) = .5000.$

b. $P(X \le 17.5) = P\left(Z \le \frac{17.5-15.0}{1.25}\right) = P(Z \le 2) = \Phi(2.00) = .9772.$

c. $P(X \ge 10) = P\left(Z \ge \frac{10-15.0}{1.25}\right) = P(Z \ge -4) = 1 - \Phi(-4.00) = 1 - .0000 = 1.$

d. $P(14 \le X \le 18) = P\left(\frac{14-15.0}{1.25} \le Z \le \frac{18-15.0}{1.25}\right) = P(-.8 \le Z \le 2.4) = \Phi(2.40) - \Phi(-0.80) = .9918 - .2119 = .7799.$

e. $P(|X - 15| \le 3) = P(-3 \le X - 15 \le 3) = P(12 \le X \le 18) = P(-2.4 \le Z \le 2.4) =$
$\Phi(2.40) - \Phi(-2.40) = .9918 - .0082 = .9836.$

38. Let X denote the diameter of a randomly selected cork made by the first machine, and let Y be defined analogously for the second machine.
$P(2.9 \le X \le 3.1) = P(-1.00 \le Z \le 1.00) = .6826$, while
$P(2.9 \le Y \le 3.1) = P(-7.00 \le Z \le 3.00) = .9987$. So, the second machine wins handily.

46.

a. $P(67 < X < 75) = P\left(\frac{67-70}{3} < \frac{X-70}{3} < \frac{75-70}{3}\right) = P(-1 < Z < 1.67) = \Phi(1.67) - \Phi(-1) = .9525 - .1587 = .7938.$

b. By the Empirical Rule, c should equal 2 standard deviations. Since $\sigma = 3$, $c = 2(3) = 6$.
We can be a little more precise, as in Exercise 42, and use $c = 1.96(3) = 5.88$.

c. Let Y = the number of acceptable specimens out of 10, so $Y \sim \text{Bin}(10, p)$, where $p = .7938$ from part **a**.
Then $E(Y) = np = 10(.7938) = 7.938$ specimens.

d. Now let Y = the number of specimens out of 10 that have a hardness of less than 73.84, so
$Y \sim \text{Bin}(10, p)$, where

$$p = P(X < 73.84) = P\left(Z < \frac{73.84-70}{3}\right) = P(Z < 1.28) = \Phi(1.28) = .8997. \text{ Then}$$

$$P(Y \le 8) = \sum_{y=0}^{8} \binom{10}{y} (.8997)^y (.1003)^{10-y} = .2651.$$

You can also compute $1 - P(Y = 9, 10)$ and use the binomial formula, or round slightly to $p = .9$ and use the binomial table: $P(Y \le 8) = B(8; 10, .9) = .265$.

60.

a. $P(X \le 100) = 1 - e^{-(100)(.01386)} = 1 - e^{-1.386} = .7499$.
$P(X \le 200) = 1 - e^{-(200)(.01386)} = 1 - e^{-2.772} = .9375$.
$P(100 \le X \le 200) = P(X \le 200) - P(X \le 100) = .9375 - .7499 = .1876$.

b. First, since X is exponential, $\mu = \frac{1}{\lambda} = \frac{1}{.01386} = 72.15$, $\sigma = 72.15$. Then
$P(X > \mu + 2\sigma) = P(X > 72.15 + 2(72.15)) = P(X > 216.45) = 1 - (1 - e^{-.01386(216.45)}) = e^{-3} = .0498$.

c. Remember the median is the solution to $F(x) = .5$. Use the formula for the exponential cdf and solve for x: $F(x) = 1 - e^{-.01386x} = .5 \Rightarrow e^{-.01386x} = .5 \Rightarrow -.01386x = \ln(.5) \Rightarrow x = -\frac{\ln(.5)}{.01386} = 50.01$ m.

62.

a. Clearly $E(X) = 0$ by symmetry, so $V(X) = E(X^2) = \int_{-\infty}^{\infty} x^2 \frac{\lambda}{2} e^{-\lambda|x|} dx = \lambda \int_0^{\infty} x^2 e^{-\lambda x} dx = \lambda \cdot \frac{\Gamma(3)}{\lambda^3} = \frac{2}{\lambda^2}$.

Solving $\frac{2}{\lambda^2} = V(X) = (40.9)^2$ yields $\lambda = 0.034577$.

b. $P(|X - 0| \le 40.9) = \int_{-40.9}^{40.9} \frac{\lambda}{2} e^{-\lambda|x|} dx = \int_0^{40.9} \lambda e^{-\lambda x} dx = 1 - e^{-40.9\lambda} = .75688$.