

13.

- a.  $P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = .10 + .15 + .20 + .25 = .70$ .
- b.  $P(X < 3) = P(X \leq 2) = p(0) + p(1) + p(2) = .45$ .
- c.  $P(X \geq 3) = p(3) + p(4) + p(5) + p(6) = .55$ .
- d.  $P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = .71$ .
- e. The number of lines not in use is  $6 - X$ , and  $P(2 \leq 6 - X \leq 4) = P(-4 \leq -X \leq -2) = P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = .65$ .
- f.  $P(6 - X \geq 4) = P(X \leq 2) = .10 + .15 + .20 = .45$ .

17.

- a.  $p(2) = P(Y = 2) = P(\text{first 2 batteries are acceptable}) = P(AA) = (.9)(.9) = .81$ .
- b.  $p(3) = P(Y = 3) = P(UAA \text{ or } AUA) = (.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$ .
- c. The fifth battery must be an  $A$ , and exactly one of the first four must also be an  $A$ . Thus,  $p(5) = P(AUUUA \text{ or } UAUUA \text{ or } UUAUA \text{ or } UUUAA) = 4[(.1)^3(.9)^2] = .00324$ .
- d.  $p(y) = P(\text{the } y^{\text{th}} \text{ is an } A \text{ and so is exactly one of the first } y - 1) = (y - 1)(.1)^{y-2}(.9)^2$ , for  $y = 2, 3, 4, 5, \dots$

30.

- a.  $E(Y) = \sum_{y=0}^3 y \cdot p(y) = 0(.60) + 1(.25) + 2(.10) + 3(.05) = .60$ .
- b.  $E(100Y^2) = \sum_{y=0}^3 100y^2 \cdot p(y) = 0(.60) + 100(.25) + 400(.10) + 900(.05) = \$110$ .

32.

- a.  $E(X) = (16)(.2) + (18)(.5) + (20)(.3) = 18.2 \text{ ft}^3$ ;  $E(X^2) = (16)^2(.2) + (18)^2(.5) + (20)^2(.3) = 333.2$ . Put these together, and  $V(X) = E(X^2) - [E(X)]^2 = 333.2 - (18.2)^2 = 1.96$ .
- b. Use the linearity/rescaling property:  $E(70X - 650) = 70\mu - 650 = 70(18.2) - 650 = \$624$ . Alternatively, you can figure out the price for each of the three freezer types and take the weighted average.
- c. Use the linearity/rescaling property again:  $V(70X - 650) = 70^2\sigma^2 = 70^2(1.96) = 9604$ . (The 70 gets squared because variance is itself a square quantity.)
- d. We cannot use the rescaling properties for  $E(X - .008X^2)$ , since this isn't a linear function of  $X$ . However, since we've already found both  $E(X)$  and  $E(X^2)$ , we may as well use them: the expected actual capacity of a freezer is  $E(X - .008X^2) = E(X) - .008E(X^2) = 18.2 - .008(333.2) = 15.5344 \text{ ft}^3$ . Alternatively, you can figure out the actual capacity for each of the three freezer types and take the weighted average.

47.

- a.  $B(4;15,.7) = .001$ .
- b.  $b(4;15,.7) = B(4;15,.7) - B(3;15,.7) = .001 - .000 = .001$ .
- c. Now  $p = .3$  (multiple vehicles).  $b(6;15,.3) = B(6;15,.3) - B(5;15,.3) = .869 - .722 = .147$ .
- d.  $P(2 \leq X \leq 4) = B(4;15,.7) - B(1;15,.7) = .001$ .
- e.  $P(2 \leq X) = 1 - P(X \leq 1) = 1 - B(1;15,.7) = 1 - .000 = 1$ .
- f. The information that 11 accidents involved multiple vehicles is redundant (since  $n = 15$  and  $x = 4$ ). So, this is actually identical to **b**, and the answer is .001.

48.  $X \sim \text{Bin}(25, .05)$

- a.  $P(X \leq 3) = B(3;25,.05) = .966$ , while  $P(X < 3) = P(X \leq 2) = B(2;25,.05) = .873$ .
- b.  $P(X \geq 4) = 1 - P(X \leq 3) = 1 - B(3;25,.05) = .1 - .966 = .034$ .
- c.  $P(1 \leq X \leq 3) = P(X \leq 3) - P(X \leq 0) = .966 - .277 = .689$ .
- d.  $E(X) = np = (25)(.05) = 1.25$ ,  $\sigma_X = \sqrt{np(1-p)} = \sqrt{25(.05)(.95)} = 1.09$ .