7.

a. $\mu_{Y \cdot 2500} = 1800 + 1.3(2500) = 5050$

b. expected change = slope = β_1 = 1.3

c. expected change = $100\beta_1$ = 130

d. expected change = $-100\beta_1 = -130$

8.

a. $\mu_{Y \cdot 2000} = 1800 + 1.3(2000) = 4400$, and $\sigma = 350$, so $P(Y > 5000) = P\left(Z > \frac{5000 - 4400}{350}\right) = P(Z > 1.71)$ = .0436.

b. Now $E(Y) = 5050$, so $P(Y > 5000) = P(Z > -.14) = .5557$.

c. $E(Y_2 - Y_1) = E(Y_2) - E(Y_1) = 5050 - 4400 = 650$, and
$V(Y_2 - Y_1) = V(Y_2) + V(Y_1) = (350)^2 + (350)^2 = 245{,}000$, so the sd of $Y_2 - Y_1$ is 494.97.
Thus $P(Y_2 - Y_1 > 0) = P\left(Z > \frac{1000 - 650}{494.97}\right) = P(Z > .71) = .2389$.

d. The standard deviation of $Y_2 - Y_1$ is 494.97 (from **c**), and
$E(Y_2 - Y_1) = 1800 + 1.3x_2 - (1800 + 1.3x_1) = 1.3(x_2 - x_1)$. Thus
$P(Y_2 > Y_1) = P(Y_2 - Y_1 > 0) = P\left(z > \frac{-1.3(x_2 - x_1)}{494.97}\right) = .95$ implies that $-1.645 = \frac{-1.3(x_2 - x_1)}{494.97}$, so $x_2 - x_1$ = 626.33.

12.

a. From the summary provided, $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-341.959231}{1585.230769} = -0.21572$ and

$\hat{\beta}_0 = \frac{\Sigma y - \hat{\beta}_1 \Sigma x}{n} = \frac{52.8 - (-.21572)(303.7)}{13} = 9.1010$. So the equation of the least squares regression line is $\hat{y} = 9.1010 - .21572x$.

Based on this equation, the predicted ammonium concentration (y) when transpiration (x) is 25 ml/h is $\hat{y}(25) = 9.1010 - .21572(25) = 3.708$ mg/L.

b. If you plug $x = 45$ into the least squares regression line, you get $\hat{y}(45) = 9.1010 - .21572(45) = -0.606$. That's an impossible ammonium concentration level, since concentration can't be negative. But it doesn't make sense to predict y at $x = 45$ from this data set, because $x = 45$ is well outside of the scope of the data (this is an example of extrapolation and its potential adverse consequences).

c. With the aid of software, SSE = $\sum_{i=1}^{13}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{13}(y_i - [9.1010 - .21572x_i])^2 = \ldots = 3.505$. Or, using the available sum of squares and a derivation similar to the one described in the section, SSE = $S_{yy} - \hat{\beta}_1 S_{xy} = 77.270769 - (-.21572)(-341.959231) = 3.505$. Either way, the residual standard deviation is $s = \sqrt{\frac{\text{SSE}}{n-2}} = \sqrt{\frac{3.505}{13-2}} = 0.564$.

The typical difference between a sample's actual ammonium concentration and the concentration predicted by the least squares regression line is about ± 0.564 mg/L.

d. With SST = $S_{yy} = 77.270769$, $r^2 = 1 - \frac{\text{SSE}}{\text{SST}} = 1 - \frac{3.505}{77.270769} = .955$. So, the least squares regression line helps to explain 95.5% of the total variation in ammonium concentration. Given that high percentage and the linear relationship visible in the scatterplot (see Exercise 4), <u>yes</u>, the model does a good job of explaining observed variation in ammonium concentration.

30.

a. $\Sigma(x_i - \bar{x})^2 = 7{,}000{,}000$, so $V(\hat{\beta}_1) = \frac{(350)^2}{7{,}000{,}000} = .0175$ and the standard deviation of $\hat{\beta}_1$ is $\sqrt{.0175} = .1323$.

b. $P(1.0 \le \hat{\beta}_1 \le 1.5) = P\left(\frac{1.0 - 1.25}{.1323} \le Z \le \frac{1.5 - 1.25}{.1323}\right) = P(-1.89 \le Z \le 1.89) = .9412$.

c. Although $n = 11$ here and $n = 7$ in **a**, $\Sigma(x_i - \bar{x})^2 = 1{,}100{,}000$ now, which is smaller than in **a**. Because this appears in the denominator of $V(\hat{\beta}_1)$, the variance is smaller for the choice of x values in **a**.

32. Let β_1 denote the true average change in runoff for each 1 m^3 increase in rainfall. To test the hypotheses H_0: $\beta_1 = 0$ versus H_a: $\beta_1 \neq 0$, the calculated t statistic is $t = \frac{\hat{\beta}_1 - 0}{s_{\hat{\beta}_1}} = \frac{.82697}{.03652} = 22.64$ which (from the printout) has an associated P-value of ~0.000. Therefore, since the P-value is so small, H_0 is rejected and we conclude that there <u>is</u> a useful linear relationship between runoff and rainfall.

A confidence interval for β_1 is based on $n - 2 = 15 - 2 = 13$ degrees of freedom. Since $t_{.025,13} = 2.160$, the interval estimate is $\hat{\beta}_1 \pm t_{.025,13} \cdot s_{\hat{\beta}_1} = .82697 \pm (2.160)(.03652) = (.748, .906)$. Therefore, we can be confident that the true average change in runoff, for each 1 m^3 increase in rainfall, is somewhere between .748 m^3 and .906 m^3.