HW4 solution

6.3.1 This is a two-sided z-test with the z statistic equal to -0.54 and the P-value equal to 0.592, which is very high. So we conclude that we do not have any evidence against H_0. A .95-confidence interval for the unknown μ is $(4.442, 5.318)$. Note that the confidence interval contains the value 5, which confirms our conclusion using the above test.

6.3.2 This is a two-sided t-test with the t statistic equal to -0.55 and the P-value equal to 0.599, which is very high. We conclude that we do not have enough evidence against H_0. A .95-confidence interval for the unknown μ is $(4.382, 5.378)$. Note that the confidence interval contains the value 5, which confirms our conclusion using the above test.

6.3.3 This is a two-sided z-test with the z statistic equal to 5.14 and the P-value equal to 0.000. So we conclude that we have enough evidence against H_0 being true. A .95-confidence interval for the unknown μ is $(63.56, 67.94)$. Note that the confidence interval does not contain the value 60, which confirms our conclusion using the above test.

6.3.7 To detect if these results are statistically significant or not we need to perform a z-test for testing $H_0 : \mu = 1$. The P-value is given by

$$p(|Z| \geqslant |(1.05-1)/(\sqrt{0.1/500})|) = 0.1138$$

So these results are not statistically significant at the 5% level, and so we have no evidence against $H_0 : \mu = 1$. Also, the observed difference of $1.05 - 1 = .05$ is well within the range that the manufacturer thinks is of practical significance. So the test has detected a small difference that is not practically significant.

6.3.10 Let θ be the probability of head on a single toss. The sample sizes required so that the margin of error (half of the length) of a $\gamma = 0.95$ confidence interval for θ is less than 0.1, 0.05, 0.01 are given by

$$n \geq \frac{1}{4}\left(\frac{z_{\frac{1+\gamma}{2}}}{\delta}\right)^2$$

So for $\delta = 0.1$ $n > 384.15$, $\delta = 0.05$ $n \geq 1536.6$ and $\delta = 0.01$ $n \geq 38415$.