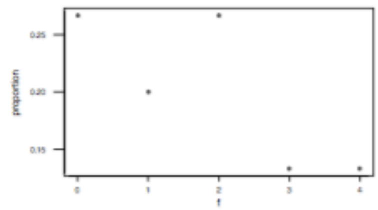
HW2 Solution

5.5.1

(a) $\hat{f}_X(0) = .2667, \hat{f}_X(1) = .2, \hat{f}_X(2) = .2667, \hat{f}_X(3) = \hat{f}_X(4) = .1333.$

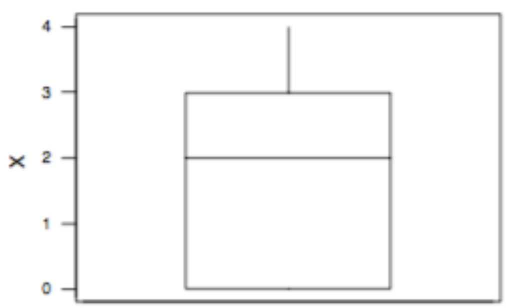
(b) $\hat{F}_X(0) = .2667, \hat{F}_X(1) = .4667, \hat{F}_X(2) = .7333, \hat{F}_X(3) = .8667, \hat{F}_X(4) = 1.000$

(c) A plot of $\hat{f}_X$ is given below.



(d) The mean is 5/3 (1.67) and the variance $s^2 = 1.952$.

(e) The median is 2 and the $IQR = 3$. The boxplot is plotted below. According to the 1.5 IQR rule, there are no outliers.



5.5.7 We have that $\psi(\mu) = x_{0.25} = \mu + \sigma_0 z_{0.25}$, where $z_{0.25}$ satisfies $\Phi(z_{0.25}) = .25$.

6.1.3 The likelihood function is given by $L(\theta \,|\, x_1, ..., x_{20}) = \theta^{20} \exp(-(20\bar{x})\,\theta)$. By the factorization theorem (Theorem 6.1.1) $\bar{x}$ is a sufficient statistic, so we only need to observe its value to obtain a representative likelihood. The likelihood function when $\bar{x} = 5.2$ is given by $L(\theta \,|\, x_1, ..., x_{20}) = \theta^{20} \exp(-20\,(5.2)\,\theta)$.

6.1.6 The likelihood function is given by

$$L(\theta \,|\, x_1, ..., x_n) = \prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i} = \theta^{\sum x_i}(1-\theta)^{n-\sum x_i} = \theta^{n\bar{x}}(1-\theta)^{n(1-\bar{x})}.$$

By the factorization theorem $\bar{x}$ is a sufficient statistic. If we differentiate $\ln L(\theta \,|\, x_1, ..., x_n) = n\bar{x} \ln \theta + n(1-\bar{x}) \ln(1-\theta)$, we get

$$(\ln L(\theta \,|\, x_1, ..., x_n))' = \frac{n\bar{x}}{\theta} - \frac{n(1-\bar{x})}{1-\theta}$$

and setting this equal to 0 gives the solution $\theta = \bar{x}$. Therefore, we can obtain $\bar{x}$ from the likelihood and we conclude that it is a minimal sufficient statistic.