Answer Keys to Homework#5

Problem 1

(a) The grand sample mean is 100. This means the treatment sum of squares is

$$SS_{Trt} = 5(3^2 + 1^2 + 6^2 + 9^2 + 2^2 + 3^2) = 700$$

The F test is equal to $(700/5)/40 = 3.5$. The critical F value for 5 and 24 degrees of freedom is 3.90. Since this is larger than the test statistic we do not reject. There is not sufficient evidence to reject the null hypothesis that the school variance is zero.

(b) The variance estimate for the error variance (between teachers) is 40 and the school variance is $(140 - 40)/5 = 20$.

(c) If $2\sigma_\tau^2 = \sigma^2$ (as is observed here) and n were 10, the value of $\lambda = \sqrt{1 + 10(.5)} = 2.45$ and the degrees of freedom error are 54. For $\alpha = .01$, this equates to $\beta = 25\%$ or power of 75%. This means that this experiment with n = 5 is very underpowered to detect such a small difference as that observed in this experiment.

(d) In class we showed that in a random effects model, the MS_{Trt} should be used in place of the MSE when computing the standard error of the grand mean. The test statistic is

$$\frac{100 - 105}{\sqrt{140/30}} = -2.31$$

We have 5 degrees of freedom so the critical t value is -2.015. Since this is smaller, we reject the null hypothesis. It does appear that the average IQ in these schools is lower than the national average.

Problem 2

You can compute Φ either using $\sum \tau_i^2$ or D. For the former, since $\sum \tau_i^2 = 2D_2$, the value of $\Phi = \sqrt{50n/30}$. For the latter, the smallest difference is $D = 5$ so $\Phi = \sqrt{25n/60}$. Given 3 treatments, the error df will be $3(n-1)$. From the chart, you need $n = 9$ observations when $\Phi = \sqrt{25n/60}$ and only $n = 4$ when $\Phi = \sqrt{50n/30}$. Notice in the notes that the value of Φ for D assumes the largest distance between means is $D/2$ so this is more conservative than specifying the means and that is why more samples are required.

Problem 3

We have 2 and 9 degrees of freedom. From the chart, $\beta = .2$ means Φ should be approximately 2.136. Solving for D gives us either 4.135 or 8.270 depending on the method.

Problem 4

(a) It followed a randomized complete block design (RCBD). Blocking strategy was applied to control the nuisance factor (ovens), and treatments (cooling temperatures) were randomly chosen for the four tiles inside each block to control for any potential factors affecting the strength.

(b) The treatment sum of squares can be calculated by

$$\begin{aligned} SS_{\text{Treatment}} &= b\sum_{i=1}^{a} \bar{y}_{i\cdot}^2 - N\bar{y}_{\cdot\cdot}^2 \\ &= 5*(5.40^2 + 5.80^2 + 10^2 + 9.80^2) - 20*7.75^2 \\ &= 92.95 \end{aligned}$$

Then the F statistic for testing the treatment effect is

$$F = \frac{SS_{\text{Treatment}}/(a-1)}{MS_E} = \frac{92.95/3}{6.275} = 4.938,$$

which is greater than $F_{0.05,3,12} = 3.49$, the 95% percentile of distribution $F_{3,12}$. Hence I conclude that there are differences among the four cooling temperatures.

(c) The critical distance for Tukey's pairwise comparisons method is

$$CD = q_{\alpha,a,(a-1)(b-1)}\sqrt{MS_E/b} = q_{0.05,4,12}\sqrt{6.275/5} = 4.705.$$

The maximum difference among the four temperature groups is $10 - 5.40 = 4.60$, which is less than the above critical difference. Hence Tukey's method detects no significant differences among the four temperature groups. As shown below, my computation is consistent with the SAS output.

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                Tukey's Studentized Range (HSD) Test for y

      NOTE: This test controls the Type I experimentwise error rate, but it
            generally has a higher Type II error rate than REGWQ.

                 Alpha                                   0.05
                 Error Degrees of Freedom                  12
                 Error Mean Square                      6.275
                 Critical Value of Studentized Range  4.19852
                 Minimum Significant Difference        4.7035

            Means with the same letter are not significantly different.

             Tukey Grouping          Mean      N    tmp
                          A        10.000      5    3
                          A         9.800      5    4
                          A         5.800      5    2
                          A         5.400      5    1
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(d) The contrasts are

$$\begin{aligned} \mathbf{C}_1 &= (1, -1, 0, 0), \\ \mathbf{C}_2 &= (0, 0, 1, -1), \\ \mathbf{C}_3 &= (1, 1, -1, -1), \end{aligned}$$

where $\mathbf{C}_1$ is used to test the difference between temperatures 5° and 10°, $\mathbf{C}_2$ to test the difference between temperatures 15° and 20°, and $\mathbf{C}_3$ to test the difference between lower temperatures (5°, 10°) and higher temperatures (15°, 20°). It's obviously that they form a complete set of orthogonal contrasts.

(e) The SAS output for testing the above contrasts is shown below.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	0.40000000	0.40000000	0.06	0.8049
C2	1	0.10000000	0.10000000	0.02	0.9016
C3	1	92.45000000	92.45000000	14.73	0.0024

Contrasts $\mathbf{C}_1$ and $\mathbf{C}_2$ are not significant, while contrast $\mathbf{C}_3$ is significant. This confirms the belief of the company that there is a jump in the strength at some temperature (12.5°) between 10° and 15°.