Answer Key to HW#4

1. (a) Let's look at the ANOVA table output from SAS.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	6026.83333	2008.94444	6.97	0.0022
Error	20	5767.00000	288.35000		
Corrected Total	23	11793.83333			

The p-value for the F-statistic is 0.0022, much less than 0.05. Hence we reject the hypothesis and conclude that there are treatment differences.

(b) Since the experiment is a balanced design, two contrasts are orthogonal to each other iff their inner product is 0. Let $\mathbf{C_1}$, $\mathbf{C_2}$ and $\mathbf{C_3}$ be respectively the contrasts for "Hormone I vs Hormone II", "Low Level vs High Level" and "Equivalence of Level". Then

$$\begin{aligned}
\mathbf{C_1^t C_2} &= 1*1+1*(-1)+(-1)*1+(-1)*(-1) = 1-1-1+1 = 0, \\
\mathbf{C_1^t C_3} &= 1*1+1*(-1)+(-1)*(-1)+(-1)*1 = 1-1+1-1 = 0, \\
\mathbf{C_2^t C_3} &= 1*1+(-1)*(-1)+1*(-1)+(-1)*1 = 1+1-1-1 = 0.
\end{aligned}$$

Hence the three contrasts are orthogonal to each other.

(c) The SAS output for contrast sums of squares and contrasts testing is as follows.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	864.000000	864.000000	3.00	0.0989
C2	1	5162.666667	5162.666667	17.90	0.0004
C3	1	0.166667	0.166667	0.00	0.9811

From the table, Contrast $\mathbf{C_1}$ is close to significant but not significant (p-value $= 0.0989$), this tells us that the average effect of hormone I and the average effect of hormone II on are not different from each other; Contrast $\mathbf{C_2}$ is very significant (p-value $= 0.0004$), this shows that the average effect for high levels of hormones and the average effect for low levels of hormones are quite different; Contrast $\mathbf{C_3}$ is not significant at all (p-value $= 0.9811$), so the difference between the high-level and low-level of hormone I is the same as that between the high-level and low-level of hormone II.

2. (a) Least significant difference (LSD):

$$CD = t_{\alpha/2,N-a}\sqrt{\frac{2MSE}{n}} = 2.12 \times 2.53 = 5.363$$

(b) Bonferroni method (for m=6 pairs):

$$CD = t_{\alpha/(2m),N-a}\sqrt{\frac{2MSE}{n}} = 3.008 \times 2.530 = 7.61$$

(c) Tukey's method:

$$CD = \frac{q_\alpha(a, N-a)}{\sqrt{2}}\sqrt{\frac{2MSE}{n}} = 2.864 \times 2.530 = 7.245$$

(d) Scheffe's method:

$$CD = \sqrt{(a-1)F_{\alpha,a-1,N-a}}\sqrt{\frac{2MSE}{n}} = 3.117 \times 2.530 = 7.887$$

(e) The smaller the minimum difference, the more power the test has. In this study, the most powerful comparison procedure is Tukey's method if all possible pairs are compared. Scheffe's procedure should be the most conservative one.

3. (a) For a), we test $H_0 : 3\mu_2 = \mu_3 + 2\mu_1$ vs. $H_1 : 3\mu_2 \neq \mu_3 + 2\mu_1$.

$$\hat{L} = \sum c_i \bar{y}_i = -3, \quad s_{\hat{L}} = \sqrt{MSE\frac{\sum c_i^2}{n_i}} = 2.79$$

test statistics:

$$t_0 = \frac{\hat{L} - L_0}{s_{\hat{L}}} = -1.076$$

$$Since\ |t_0| < t(24), we\ accept\ H_0$$

For b), we test $H_0 : \mu_1 = \mu_3$ vs. $H_1 : \mu_1 \neq \mu_3$.

$$\hat{L} = \sum c_i \bar{y}_i = 3, \quad s_{\hat{L}} = \sqrt{MSE\frac{\sum c_i^2}{n_i}} = 1.05$$

test statistics:

$$t_0 = \frac{\hat{L} - L_0}{s_{\hat{L}}} = 2.846$$

$$Since\ |t_0| > t(24), we\ reject\ H_0$$

For c), we test $H_0 : \mu = 3$ vs. $H_1 : \mu \neq 3$.

$$\hat{L} = \frac{\hat{\mu_1} + \hat{\mu_2} + \hat{\mu_3}}{3} = \frac{4}{3}, \quad s_{\hat{L}} = \sqrt{MSE\frac{\sum c_i^2}{n_i}} = 0.43$$

test statistics:

$$t_0 = \frac{\hat{L} - L_0}{s_{\hat{L}}} = -3.875$$

$$Since\ |t_0| > t(24), we\ reject\ H_0$$

(b) Linear combinations a) and b) are contrasts because sum of the coefficents equals zero.

(c) Orthogonal contrast for balanced eperiment:$\sum_{i=1}^{a} c_i d_i = 0$

contrast a) and b):$-2 \times 1 + 3 \times 0 - 1 \times (-1) = -1$

contrast a) and c):$-2 \times (1/3) + 3 \times (1/3) - 1 \times (1/3) = 0$

contrast b) and c):$1 \times (1/3) + 0 \times (1/3) - 1 \times (1/3) = 0$

Both contrast a),c) and contrast b),c) are orthogonal.

(d) The two tests are different.The reason is that variance of two linear cominations are different.

$$var(\hat{\mu}) = \frac{\sigma^2}{n} \quad var(\hat{\mu_1} - \hat{\mu_2}) = \frac{2\sigma^2}{n}$$

4. (a) Usual ANOVA shows that $F_0 = 21.31$ and P-value is less than 0.001. We reject null hypothesis and conclude that difference exists among the treatment effects.

(b) Residual plots indicate that constant variance assumption might be invalid. Both Levene's test and bartlett's test report P-value less than 5%, which implies that some remedy is in order.

(c) Using the SAS file for approximate Box-Cox transformation, one has

$$\log s_i = -.714 + .835 \log \bar{y}_{i.},$$

and $\hat{\beta} = .835$, so the possible (variance stablizing) transformation is

$$Y' = Y^{1-\hat{\beta}} = Y^{.165}$$

Since $\hat{\beta}$ is approximately 1.00, a more meaningful transformation should be

$$Y' = \log(Y).$$

(d) Use the sas file for the exact Box-Cox transformation, SS_E is minimized at $\lambda = .25$. The transformation is

$$Y' = Y^{1/4}$$

In fact, $\lambda = 1/4$ and $\lambda = 0$ might not be different statistically, so both transfomations can be used.

(e) Apply ANOVA to the transformed responses. Residual plots and formal tests show that the violation of constant variance assumption has been corrected.

5. Apply usual ANOVA first to the data. QQ plot reveals some departure from normality, formal tests (Shapiro-Wilk's test) report p-values less than 5%. This implies that the normality assumption is not valid and the result from ANOVA are questionable. Hence a nonparametric procedure is called for. Use `PROC NPAR1WAY` to perform the Kruskal-Wallis test. The conclusion from Kruskal-Wallis is consistent with that from ANOVA in this problem.

6. (a) The random effects model is appropriate since both the needles and the rows are randomly seleted.

(b)

Source	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Model	9	1299.725	144.413889	2.68	0.0203
Error	30	1614.25	53.808333		
Total	39	29313.975			

$$\hat{\sigma}^2 = MSE = 53.808333$$

$$\hat{\sigma}_\tau^2 = \frac{MS(Trt) - MSE}{n} = 22.6514$$

(c)

$$\rho IC = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2} = \frac{22.65}{22.65 + 53.81} = 0.2962$$

29.62% of the overall variation in stomata number per centimeter is due to the needle.

(d) The 95% CI for the ratio is $[\frac{L}{L+1}, \frac{U}{U+1}]$ where,

$$L = \frac{1}{n}(\frac{MSTR}{MSE \times F_{1-\alpha/2}} - 1) = 0.0106, \quad U = \frac{1}{n}(\frac{MSTR}{MSE \times F_{\alpha/2}} - 1) = 2.1389$$

Thus the 95% confidenc interval for the ratio is $[0.01049, 0.6814]$

(e)

$$\hat{\mu} = \bar{Y}_{..} = 130.475$$

$$\bar{Y}_{..} \pm t(1-\alpha/2, r-1)\sqrt{\frac{MSTR}{rn}} = 130.475 \pm 2.042\sqrt{\frac{144.413889}{40}}$$

$$95\% CI: \quad [126.60, 134.35]$$