STAT 514 Homework#3
(Due Thursday 09/17/15 BEFORE CLASS)

1. To determine whether waste discharged by a chemical plant is polluting the local river, the EPA plans to take water samples both upstream and downstream from the discharge site and measure the concentration level (ppm) of the suspected chemical pollutant. A two-sample t-test will be used to assess $H_0 : \mu_{\text{Down}} = \mu_{\text{Up}}$ vs $H_A : \mu_{\text{Down}} > \mu_{\text{Up}}$. You have been hired to determine the number of samples to take at each location and will use a modified version of `tpower.sas` to do these calculations. Since they are interested in a one-sided alternative, you need to eliminate the lower critical value (rlow) and compute the upper critical value using $1 - \alpha$ instead of $1 - \alpha/2$. This means the loop for sample size should be changed to

```
/* Sample size calculation code for the > alternative */
do n=2 to 11 by 1;
   df=2*(n-1); nc = delta/(sigma*sqrt(2/n));
   rhigh = tinv(1-alpha,df);
   p=1-probt(rhigh,df,nc); output;
end;
```

(a) What sample size is needed to detect a change in true concentrations of 5 ppm 90% of the time when $\alpha = .05$ and $\sigma = 4.0$ (NOTE: You may need to change the range for n)?

(b) Suppose instead they want to detect a change in true concentrations of 5 ppm 95% of the time. What value of n is needed?

(c) Now consider that they want to detect a change in true concentrations of 5 ppm 95% of the time but set $\alpha = .10$. What value of n is needed?

(d) Using the results of (a), (b), and (c), what happens to n when all else is kept constant but the desired power is increased? What happens to n when all is kept constant but the α is increased?

2. Bjorn Studley selects fourteen judges (a.k.a. chumps off the street) and randomly allocates a can of Schlitz or Billy Beer to each of them. He then asks each judge to rate the taste of the beer on a 1 (yuck) to 10 (yum) scale. The results are

Schlitz	2	4	1	7	4	3	1	5
Billy	4	3	6	8	7	5		

(a) Perform the appropriate analysis stating the hypotheses and assumptions (use $\alpha = .05$);

(b) Suppose the experimenter wanted to detect differences in taste of at least 2 units (with 95% power). Using the estimated pooled variance from part `a`), what is the power of this experiment at $\delta = 2$? To answer this, use the following SAS commands replacing "???" with the estimated standard deviation.

```
data new1; n1=8; n2=6; sigma=???; delta=2; alpha=0.05;
    df = n1+n2-2; nc=delta/(sigma*(sqrt(1/n1+1/n2)));
    rlow = tinv(alpha/2,df); rhigh=tinv(1-alpha/2,df);
    p=1-probt(rhigh,df,nc)+probt(rlow,df,nc);
proc print data=new1; run;
```

(c) Besides increasing sample size, describe a design improvement that would likely result in a reduction of the error variance (HINT: we've only talked about one other design besides the two-sample t-test).

3. An experiment is done where 5 trts are randomly and equally assigned to 30 experimental units. Suppose that

$$\sum\sum(y_{ij} - \overline{y}_{..})^2 = 1230, \qquad n\sum(\overline{y}_{i.} - \overline{y}_{..})^2 = 650$$

Write out the ANOVA table and state whether there is at least one treatment effect different from zero at the .05 level.

4. A vendor submits lots of fabric to a textile manufacturer. The manufacturer wants to know if the lot average breaking strength exceeds 200 psi. if so, she wants to accept the lot. Past experience indicates that a reasonable value for the variance of breaking strength is 100 $(\text{psi})^2$. The hypotheses to be tested are

$$H_0 : \mu = 200 \text{ vs. } H_1 : \mu > 200.$$

The manufacturer decides to randomly select a number of specimens, measure their breaking strengths and test the hypotheses with $\alpha = 5\%$. Suppose she wants to guarantee that average breaking strength 210 or higher should be detected with probability at least 95%. What is the minimum number of specimens she should check? (Hint: because the distribution of the sample mean under $\mu = 210$ is normal, you do not need to generate the O.C. curves to determine the sample size).