Answer Keys to Homework#3

1. (a) A sample size of 12 is needed.
(b) A sample size of 15 is needed.

Obs	alpha	sigma	delta	n	df	nc	rhigh	p
1	0.05	4	5	11	20	2.93151	1.72472	0.88204
2	0.05	4	5	12	22	3.06186	1.71714	0.90671
3	0.05	4	5	13	24	3.18689	1.71088	0.92650
4	0.05	4	5	14	26	3.30719	1.70562	0.94231
5	0.05	4	5	15	28	3.42327	1.70113	0.95486

(c) A sample size of 12 is needed.

Obs	alpha	sigma	delta	n	df	nc	rhigh	p
1	0.1	4	5	11	20	2.93151	1.32534	0.94388
2	0.1	4	5	12	22	3.06186	1.32124	0.95744
3	0.1	4	5	13	24	3.18689	1.31784	0.96781
4	0.1	4	5	14	26	3.30719	1.31497	0.97571
5	0.1	4	5	15	28	3.42327	1.31253	0.98171

(d) When all else is kept constant, we need to increase the sample size n to increase the power; and we will need a smaller sample size n for an increased α.

2.(a) Hypothesis:

$$H_0 : \mu_s = \mu_b \quad H_1 : \mu_s \neq \mu_b$$

$$S^2_{pool} = \frac{7 \times 2.07^2 + 5 \times 1.87^2}{8 + 6 - 2} = 3.95$$

Test statistics:

$$t_0 = \frac{\bar{y_s} - \bar{y_b}}{s_{pool}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{3.375 - 5.5}{\sqrt{3.95(1/8 + 1/6)}} = -1.98$$

From the studentized distribution table,$t_{0.975,12} = 2.17, t_{0.025,12} = -2.17, |t_0| < 2.17$,thus we would not reject null hypothesis and conclude there is not sufficient evidenc to stat the taste of two beers are different.
(b) The power of this experiment at $\delta = 2$ is 0.40230.

Obs	n1	n2	sigma	delta	alpha	df	nc	rlow	rhigh	p
1	8	6	1.99	2	0.05	12	1.86094	-2.17881	2.17881	0.40230

(c) A paired two-sample design would likely result in a reduction of the error variance.

3.
Test $H_0 : \tau_1 = \tau_2 = ... = \tau_5 = 0$ vs. H_1 : at least one $\tau_i \neq 0$.

	DF	Sum Sq	Mean Sq	F value
Between	4	650	162.5	7.00
Within	25	580	23.2	
Total	29	1230		

Since test statistics $F_0 = \frac{MS_{Trt}}{MSE} = 7.00 > F_{0.05,4,25} = 2.76$, we reject H_0, we conclude that there is at least one treatment effect different from zero at the 0.05 level.

4. Because the population variance is known to be $\sigma^2 = 100$, only z-test is needed. The test statistic is

$$Z = \frac{\bar{X} - 200}{\sigma/\sqrt{n}}.$$

The decision rule is that

$$\text{Reject } H_0, \text{ if } Z > z_{0.05} = 1.645$$

or equivalently

$$\text{Reject } H_0, \text{ if } \bar{X} > 200 + 1.645\frac{10}{\sqrt{n}}.$$

The probability of type II error is

$$\beta = P(\text{ accept } H_0 \mid H_1) = P(\bar{X} \leq 200 + 1.645\frac{10}{\sqrt{n}} \mid H_1)$$

In particular, the manufacturer wants to control β for $\mu \geq 210$. She requires that

$$\beta = P(\bar{X} \leq 200 + 1.645\frac{10}{\sqrt{n}} \mid \mu = 210) \leq 5\%.$$

Under $\mu = 210$, $\bar{X}$ follows $N(210, \frac{10}{\sqrt{n}})$. So,

$$\beta = P(\frac{\bar{X} - 210}{10/\sqrt{n}} \leq -\sqrt{n} + 1.645) \leq 0.05.$$

Hence,

$$-\sqrt{n} + 1.645 \leq -1.645,$$

and

$$n \geq (1.645 * 2)^2 = 10.8.$$

She has to check at least 11 lots.