

Answer Keys to Homework#2

1.

$$H_0 : \mu_B = \mu_A \text{ vs } H_1 : \mu_B \neq \mu_A$$

a) There are $\binom{6}{3} = 20$ ways to allocate 3A's and 3B's to the six plants. The average differences between B and A are

$-2.67, -2, -1.33, -1.33, -1.33, -.67, -.67, -.67, 0, 0, 0, 0, .67, .67, .67, 1.33, 1.33, 1.33, 2, 2.67,$

each equally likely. The distribution can be represented by a histogram or other graphics.

Or, let δ denote the mean difference.

$$p(\delta) = \begin{cases} -2.67 & 1/20 \\ -2 & 1/20 \\ -1.33 & 3/20 \\ -0.67 & 3/20 \\ 0 & 4/20 \\ 0.67 & 3/20 \\ 1.33 & 3/20 \\ 2 & 1/20 \\ 2.67 & 1/20 \end{cases}$$

b) $\delta_{obs}=1.33$. Considering that it is a two-sided t test,

$$P - \text{value} = P(\delta \geq 1.33 \text{ or } \delta \leq -1.33) = \frac{10}{20} = .50$$

c) $s_A^2 = 3$, $s_B^2 = 2.33$, $s_{pool}^2 = 2.67$, and $\bar{y}_B - \bar{y}_A = 1.33$. The observed test statistic

$$T_{obs} = \frac{\bar{y}_B - \bar{y}_A}{s_{pool} \sqrt{1/3 + 1/3}} = 1$$

$$P - \text{value} = P(T \leq -1 \text{ or } T \geq 1 \mid t(4)) = .374.$$

d) Use the usual significance level α , we will accept H_0 in both b) and c). Recall that t tests rely on strong assumptions, nonetheless they are good approximations to randomization tests even when sample sizes are small.

2. Both typists and manuscripts should be treated as blocks. In all, we have 12 blocks. Within each block, all treatments (three types of keyboards) should be applied. The order of using the keyboards should be randomized. If we want to eliminate the learning effects entirely, balanced randomization should be used. Advantages of randomization and blocking refer to the notes.

3.

a) negative.

b) omitted.

c) Let

$$\bar{y}_B = \frac{\sum_{i=1}^6 y_{iB}}{6} \text{ and } \bar{y}_A = \frac{\sum_{i=1}^6 y_{iA}}{6}$$

$\bar{y}_B - \bar{y}_A$ can be used to estimate $\tau_B - \tau_A$. (A more complex estimator using the linear model approach, but not required for this question)

d) If Design 1 were used,

$$E(\bar{y}_B - \bar{y}_A) = \tau_B - \tau_A + \eta.$$

If Design 2 were used,

$$E(\bar{y}_B - \bar{y}_A) = \tau_B - \tau_A + \frac{1}{3}\eta.$$

It is clear that the impact of the learning effect on the estimate is reduced by applying randomization.

d). If balanced randomization were applied, we have

$$E(\bar{y}_B - \bar{y}_A) = \tau_B - \tau_A.$$

That is, $\bar{y}_B - \bar{y}_A$ becomes an unbiased estimator for $\tau_B - \tau_A$. e). omitted.