Answer Keys to Homework#1

1.

(a) H_0: $\mu \le 225$ (or H_0: $\mu = 225$) vs H_1: $\mu > 225$

(b)

$$\bar{y} = 241.50$$
$$S^2 = 146202 / (16 - 1) = 9746.80$$
$$S = \sqrt{9746.8} = 98.73$$
$$t_o = \frac{\bar{y} - \mu_o}{\frac{S}{\sqrt{n}}} = \frac{241.50 - 225}{\frac{98.73}{\sqrt{16}}} = 0.67$$

Since $t_{0.05,15} = 1.753$, we fail to reject H_0

(c) P=0.26

(d) The 95% confidence interval is $\bar{y} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$

$$241.50 - (2.131)\left(\frac{98.73}{\sqrt{16}}\right) \le \mu \le 241.50 + (2.131)\left(\frac{98.73}{\sqrt{16}}\right)$$

$$188.9 \le \mu \le 294.1$$

2.

$E(Z_i)=E(X)+E(Y_i)=5+0=5$

$Var(Z_i)=Var(X)+Var(Y_i)=1+1/2=3/2$

$E(\bar{Z})=E(X+\bar{Y})=E(X)+E(\bar{Y})=5+0=5$

$Var(\bar{Z})=Var(X+\bar{Y})=Var(X)+Var(\bar{Y})=1+0.5/10=1.05$

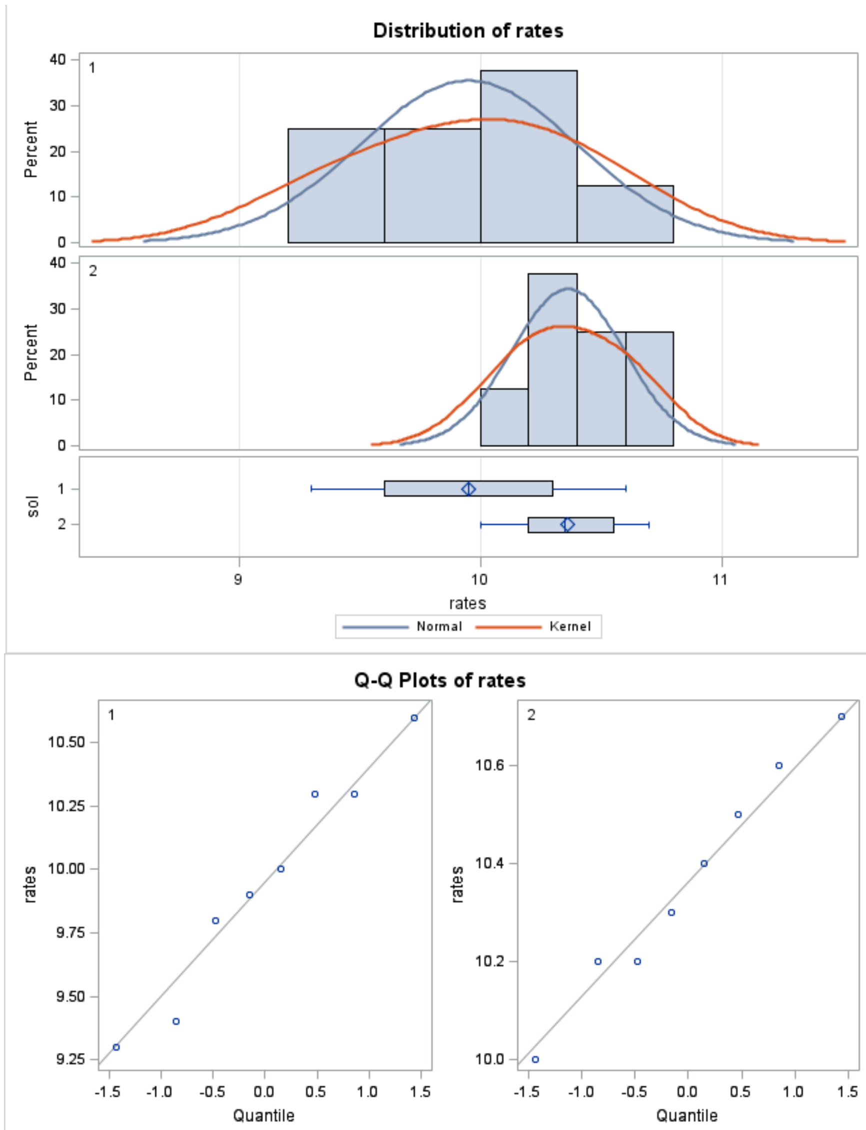
3. Test H_0: $\mu_1-\mu_2 \le 3$ (or H_0: $\mu_1-\mu_2 = 3$) vs. H_1: $\mu_1-\mu_2 > 3$. t=(1.4167-3)/4.1083=-0.3854. p-value=0.6482. No evidence shows that formulation 1 exceeds formulation 2 by at least 3 °F.

4.

(a) t = -0.4125/0.1792 = -2.30 (df=14). So p-value = 0.0372, the solutions do not have the same mean etch rate.

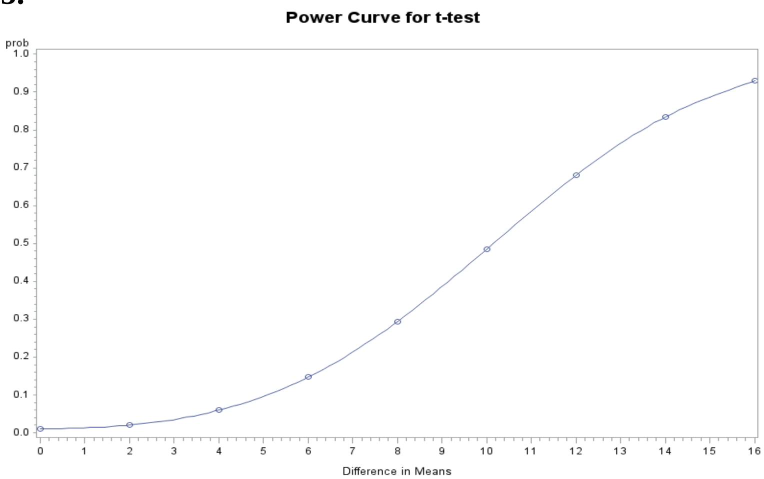
(b) The 95% confidence interval on the difference in mean etch rate is (-0.7969, -0.0281).

(c) Use PROC TTEST in SAS to output the following plots.



The normality assumptions are valid but the equal variances assumption may not be valid.

5.



6. (2.17) The viscosity of a liquid detergent is supposed to average 800 centistokes at 25° C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is $\sigma = 25$ centistokes.

(a) State the hypotheses that should be tested.

H_0: $\mu = 800$ $\quad\quad$ H_1: $\mu \neq 800$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{812 - 800}{\frac{25}{\sqrt{16}}} = \frac{12}{\frac{25}{4}} = 1.92$$

Since $z_{\alpha/2} = z_{0.025} = 1.96$, do not reject.

(c) What is the P-value for the test? $\quad P = 2(0.0274) = 0.0549$

(d) Find a 95 percent confidence interval on the mean.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$812 - (1.96)(25/4) \leq \mu \leq 812 + (1.96)(25/4)$$
$$812 - 12.25 \leq \mu \leq 812 + 12.25$$
$$799.75 \leq \mu \leq 824.25$$

7. (2.43) Since $\quad \dfrac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \sim F_{n_2-1,n_1-1}$

$$P\left\{F_{1-\alpha/2,n_2-1,n_1-1} \leq \frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \leq F_{\alpha/2,n_2-1,n_1-1}\right\} = 1-\alpha \quad \text{or}$$

$$P\left\{\frac{S_1^2}{S_2^2}F_{1-\alpha/2,n_2-1,n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2}F_{\alpha/2,n_2-1,n_1-1}\right\} = 1-\alpha$$

8.

(a) The degrees of freedom for each of the following are:

(a-1) factor A: 2
(a-2) factor B: 4
(a-3) interaction AB: 8
(a-4) model: 14
(a-5) error: 43

(b) Since MSA=SSA/2=113 and MSE=SSE/43=14.0465, we have
F=MSA/MSE=113/14.0465=8.0447 with 2, 43 df.

9.

MSA=SSA/2=100/2=50
MSB=SSB/1=60
MSAB=SSAB/2=20

(a) F=MSA/MSE=50/10=5
(b) F=MSB/MSAB=60/20=3
(c) F=MSAB/MSE=20/10=2