

Solution for Hw#1

1. (2.17) The viscosity of a liquid detergent is supposed to average 800 centistokes at 25° C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is $\sigma = 25$ centistokes.

(a) State the hypotheses that should be tested.

$$H_0: \mu = 800 \quad H_1: \mu \neq 800$$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{812 - 800}{\frac{25}{\sqrt{16}}} = \frac{12}{\frac{25}{4}} = 1.92 \quad \text{Since } z_{\alpha/2} = z_{0.025} = 1.96, \text{ do not reject.}$$

(c) What is the P -value for the test? $P = 2(0.0274) = 0.0549$

(d) Find a 95 percent confidence interval on the mean.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{y} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$812 - (1.96)(25/4) \leq \mu \leq 812 + (1.96)(25/4)$$

$$812 - 12.25 \leq \mu \leq 812 + 12.25$$

$$799.75 \leq \mu \leq 824.25$$

2. (2.22) The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

$$H_0: \mu = 225 \quad H_1: \mu > 225$$

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

$$\bar{y} = 241.50$$
$$S^2 = 146202 / (16 - 1) = 9746.80$$

$$S = \sqrt{9746.8} = 98.73$$

$$t_o = \frac{\bar{y} - \mu_o}{\frac{S}{\sqrt{n}}} = \frac{241.50 - 225}{\frac{98.73}{\sqrt{16}}} = 0.67$$

since $t_{0.05,15} = 1.753$; do not reject H_0

Minitab Output

T-Test of the Mean

Test of mu = 225.0 vs mu > 225.0

Variable	N	Mean	StDev	SE Mean	T	P
Hours	16	241.5	98.7	24.7	0.67	0.26

T Confidence Intervals

Variable	N	Mean	StDev	SE Mean	95.0 % CI
Hours	16	241.5	98.7	24.7	(188.9, 294.1)

(c) Find the P -value for this test. $P=0.26$

(d) Construct a 95 percent confidence interval on mean repair time.

The 95% confidence interval is $\bar{y} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{y} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}$

$$241.50 - (2.131) \left(\frac{98.73}{\sqrt{16}} \right) \leq \mu \leq 241.50 + (2.131) \left(\frac{98.73}{\sqrt{16}} \right)$$

$$188.9 \leq \mu \leq 294.1$$

3.

(a) $E(Z_i) = E(X) + E(Y_i) = 5 + 0 = 5$

(b) $\text{Var}(Z_i) = \text{Var}(X) + \text{Var}(Y_i) = 1 + 1/2 = 3/2$

(c) $E(\bar{Z}) = E(X + \bar{Y}) = E(X) + E(\bar{Y}) = 5 + 0 = 5$

(d) $\text{Var}(\bar{Z}) = \text{Var}(X + \bar{Y}) = \text{Var}(X) + \text{Var}(\bar{Y}) = 1 + 1/(2 \cdot 10) = 21/20$

4. (2.36) Refer to the data in problem 2-35. Do the data support a claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2 by at least 3 °F?

No, formulation 1 does not exceed formulation 2 by at least 3 °F.

Minitab Output

Two-Sample T-Test and CI: Form1, Form2

Two-sample T for Form 1 vs Form 2

	N	Mean	StDev	SE Mean
Form 1	12	194.5	10.2	2.9

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Form 2  12    193.08    9.95    2.9
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Difference = mu Form 1 - mu Form 2
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Estimate for difference:  1.42
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95% lower bound for difference: -5.64
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T-Test of difference = 3 (vs >): T-Value = -0.39  P-Value = 0.648  DF = 22
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Both use Pooled StDev = 10.1
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5. (2.37) In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic of this process. Two different etching solutions are being evaluated. Eight randomly selected wafers have been etched in each solution and the observed etch rates (in mils/min) are shown below:

Solution 1		Solution 2	
9.9	10.6	10.2	10.6
9.4	10.3	10.0	10.2
10.0	9.3	10.7	10.4
10.3	9.8	10.5	10.3

- (a) Do the data indicate that the claim that both solutions have the same mean etch rate is valid? Use $\alpha = 0.05$ and assume equal variances.

No, the solutions do not have the same mean etch rate. See the Minitab output below.

Minitab Output

Two Sample T-Test and Confidence Interval

Two-sample T for Solution 1 vs Solution 2

	N	Mean	StDev	SE Mean
Solution 1	8	9.950	0.450	0.16
Solution 2	8	10.363	0.233	0.082

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Difference = mu Solution 1 - mu Solution 2
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Estimate for difference: -0.413
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95% CI for difference: (-0.797, -0.028)
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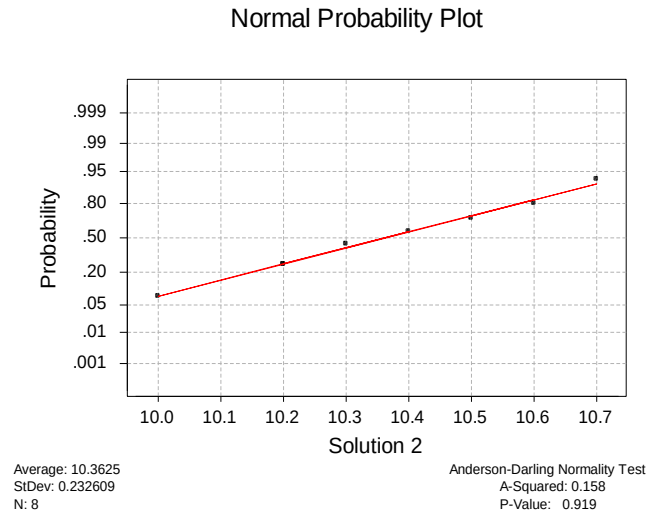
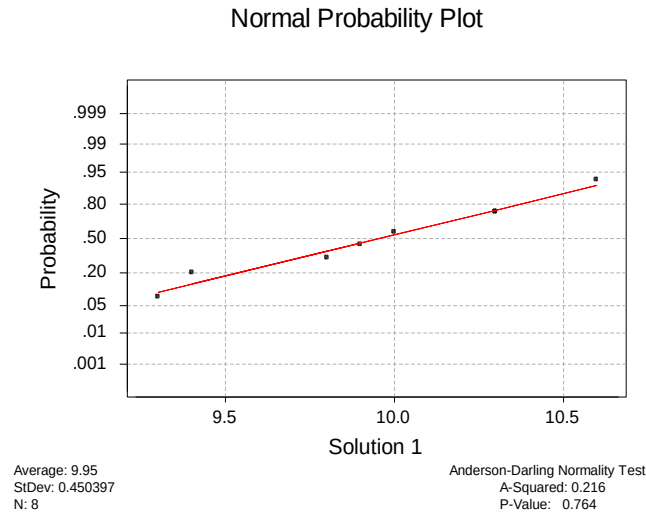
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T-Test of difference = 0 (vs not =): T-Value = -2.30  P-Value = 0.037  DF = 14
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Both use Pooled StDev = 0.358
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- (b) Find a 95% confidence interval on the difference in mean etch rate.

From the Minitab output, -0.797 to -0.028.

- (c) Use normal probability plots to investigate the adequacy of the assumptions of normality and equal variances.



Both the normality and equality of variance assumptions are valid.

- 6. (2.43)** Develop Equation 2-50 for a $100(1 - \alpha)$ percent confidence interval for the ratio σ_1^2 / σ_2^2 , where σ_1^2 and σ_2^2 are the variances of two normal distributions.

$$\frac{S_2^2 / \sigma_2^2}{S_1^2 / \sigma_1^2} \sim F_{n_2-1, n_1-1}$$

$$P\left\{F_{1-\alpha/2, n_2-1, n_1-1} \leq \frac{S_2^2 / \sigma_2^2}{S_1^2 / \sigma_1^2} \leq F_{\alpha/2, n_2-1, n_1-1}\right\} = 1 - \alpha \quad \text{or}$$

$$P\left\{\frac{S_1^2}{S_2^2} F_{1-\alpha/2, n_2-1, n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} F_{\alpha/2, n_2-1, n_1-1}\right\} = 1 - \alpha$$

- 7. (a)** the degrees of freedom for each of the following are:

- (a-1) factor A: 2
- (a-2) factor B: 4
- (a-3) interaction AB: 8
- (a-4) model: 14
- (a-5) error: 43

(b) Since $MSA = SSA/2 = 113$ and $MSE = SSE/43 = 14.0465$, we have $F = MSA/MSE = 113/14.0465 = 8.0447$ with 2, 43 df.

8.

$$MSA = SSA/2 = 100/2 = 50$$

$$MSB = SSB/1 = 60$$

$$MSAB = SSAB/2 = 20$$

- (a) $F = MSA/MSE = 50/10 = 5$
- (b) $F = MSB/MSAB = 60/20 = 3$
- (c) $F = MSAB/MSE = 20/10 = 2$

9.

(a) CONTRAST A 0.5 0.5 -1 A*B 0.5 0 0.5 0 -1 0

(b) $(9 \cdot (0.5^2 + 0.5^2 + 1^2)/4)^{0.5} = 1.8371$

10.

$$S^2 = MSE = 30$$

$$S_A^2 = (MSA - MSE)/8 = (100 - 30)/8 = 8.75$$