Statistics 512

Major Exam 1

Monday, October 9, 2013

NAME:

Materials permitted:
This exam is open-book and open-note. Anything on paper is allowed. Calculators are allowed. HOWEVER, computers, phones, or any devices capable of wireless communications are not permitted.

Please

- do not open the exam until I say you may
- do not cheat on this exam (do not copy from or communicate with any other person)
  Sign Here:
- write your solution clearly and legibly so that I can follow it
- cross out your mistakes; do not erase large quantities of work hand it in
- you have **60 minutes**, at which time all papers will be collected.
- unless otherwise stated, use $\alpha = 0.05$.

A SLR model fit to 25 data points gave $b_0 = 3,\ b_1 = -2,\ \bar{X} = 3,\ s_X = 2,\ s_Y = 5,\ \sum e_i^2 = 230$.

1. [4 points] Find the residual when the response variable is -5 and the explanatory variable is 5. Is the point above or below the line?

   Solution: $\hat{Y}_i = 3 - 2 \times 5 = -7 \quad \Rightarrow \quad e_i = -5 + 7 = 2$, and the point is above the line.

2. [4 points] Compute root MSE for this model.

   Solution: $s = \sqrt{MSE} = \sqrt{\frac{230}{23}} = \sqrt{10} \approx 3.16$.

3. [4 points] Calculate the correlation between X and Y. Interpret this value.

   Solution: $r = -2 \cdot \frac{2}{5} = -0.8$. There is a strong negative association (correlation) between X and Y. If you use $s_Y = 5.1$, then $r = -0.79$

4. [5 points] What is SSR for this model?

   Solution: $SSR = SST - SSE$, but $SST = \frac{SSR}{R^2} = \frac{SSE}{1-R^2} = 639 \quad \Rightarrow \quad SSR = 409$. If you use $s_Y = 5.1$, then $SSR = 370$.

5. [4 points] Construct a 95% confidence interval for the slope. [Hint: find SS_X first].

   Solution: $b_1 \pm t_{23} \times s\{b_1\}$. But, $SS_X = (n-1) \cdot s_X^2 = 96 \quad \Rightarrow \quad s\{b_1\} = 0.323$. The C.I. is

   $$-2 \pm 2.069 \times 0.323 \quad \Rightarrow \quad (-2.67, -1.33).$$

6. [4 points] Test the hypothesis $\mathrm{H}_a:\ \rho \neq 0$.

   Solution: $\mathrm{H}_0:\ \rho = 0$.
   $t_s = t_{23} = \frac{r\sqrt{n-2}}{1-r^2} = -6.2$.
   $t_c = 2.069$.
   Since $|t_s| > t_c$ we reject H_0.
   or $t_s = \frac{b_1}{s\{b_1\}} = \frac{-2}{0.323} = -6.2$, which gives the same conclusion.
   or Since 0 falls outside the 95% C.I. for β_1, we reject H_0.

7. [5 points] You suspect the slope is less than -1.5, test this claim.

   Solution: $\mathrm{H}_0:\ \beta_1 = -1.5 \quad vs \quad \mathrm{H}_a:\ \beta_1 < -1.5$.
   $t_s = t_{23} = \frac{-2+1.5}{0.323} = -1.55$.
   $t_c = -1.714$.
   Since $t_s < t_c$ we fail to reject H_0.

8. [4 points] Find a 95% interval for the mean response when the explanatory is 5.

   Solution: $\hat{Y} \pm t_{23} \times s \times \sqrt{\frac{1}{n} + \frac{(X_h - \bar{X})^2}{SS_X}}$

   $$-7 \pm 2.069 \times \sqrt{10} \times \sqrt{\frac{1}{25} + \frac{(5-3)^2}{96}} \quad \Rightarrow \quad (-8.87, -5.13).$$

9. [4 points] You are running an experiment and would like to calibrate the explanatory variable, find a calibrated value for the explanatory when the response is -5.

   Solution: $\hat{X} = -\frac{b_0}{b_1} + \frac{1}{b_1} \cdot Y_h = 1.5 + \frac{5}{2} = 4$.

10. [2 points] If the power for detecting a change in β_1 is 0.07, what can you say about the α level?

    Solution: $\alpha \leq 0.07$.

Good Luck! ☺