Statistics 512: Problem Set 4

Due Thursday, March 8, 2018 11:59 PM

Important Note – Every graph or plot you create should have your name printed as a subtitle. Consequently, any graph with no name will result in a **20% points loss**. Also, please attach your code at the end; any homework with no code provided will result in a **50% points loss** on the entire assignment.

1. Consider the following SAS output giving 5 confidence intervals for the mean of Y. If you wanted to guarantee that **joint** coverage of the five confidence intervals was at least 95%, what confidence level would you use when forming each interval, using the Bonferroni correction? Compute this adjusted confidence interval for the mean of Y when X = 4. (Note that some observations have been omitted from the output.)

		Analys	is of V	arianc	е					
		:	Sum of		Mean					
Source		DF S	quares	Sq	uare	F	Value	Pr > F		
Model		1	16183	1	6183	80	5.62	<.0001		
Error		16 321	.39597	20.0	8725					
Correcte	ed Total	17	16504							
Root MSE	Ξ	4.48188	R-S	quare	0.9805					
Depender	nt Mean (64.00000	Adj	R-Sq	0.9793					
Coeff Va	ar	7.00294								
Paramete	er Estima	ates								
]	Paramete:	r Stan	dard						
Variable	e DF	Estimat	e E	rror	t Valu	е	Pr >	t		
Intercep	ot 1	-2.3221	5 2.5	6435	-0.9	1	0.3	786		
x	1	14.7382	6 0.5	1926	28.3	8	<.0	001		
Output S	Statisti	cs								
	Dep Va	r Predi	cted	Std	Error					
Obs x		y Va	alue	Mean P	redict		95% C	L Mean	Residu	al
35	78.000	0 71.3	3691		1.0878	6	9.0630	73.6752	6.63	09
4 1	10.000	0 12.4	4161		2.1021		7.9598	16.8724	-2.41	61
6 4	62.000	56.	5309		1.0878	5	4.3248	58.9370	5.36	91
8 3	39.000	0 41.3	3926		1.3125	3	9.1103	44.6750	-2.89	26
10 2	33.000	0 27.	1544		1.6737	2	3.6064	30.7024	5.84	56

2. Based on the following small data set, construct the design matrix, \mathbf{X} , its transpose $\mathbf{X'}$, and the matrices $\mathbf{X'X}$, $(\mathbf{X'X})^{-1}$, $\mathbf{X'Y}$, and $\mathbf{b} = (\mathbf{X'X})^{-1}\mathbf{X'Y}$. (Chapter 5 in the book discusses finding the inverse of a matrix.)

X	Y
0	1
1	4
2	7
3	9
4	10
5	8

For the following 5 problems, consider the data given in the file CH06PR18.DAT, which describes a data set (n = 24) used to evaluate the relation between intermediate and senior level annual salaries of bachelor's and master's level mathematicians (Y, in thousand dollars) and an index of work quality (X_1) , number of years of experience (X_2) , and an index of publication success (X_3) .

- 3. Run the multiple linear regression with quality, experience, and publications as the explanatory variables and salary as the response variable. Summarize the regression results by giving the fitted regression equation, the value of R^2 , and the results of the significance test for the null hypothesis that the three regression coefficients for the explanatory variables are all zero (give null and alternative hypotheses, test statistic with degrees of freedom, *p*-value, and brief conclusion in words).
- 4. Give 85% confidence intervals (do not use a Bonferroni correction) for regression coefficients of quality, experience, and publications based on the multiple regression. Describe the results of the hypothesis tests for the individual regression coefficients (give null and alternative hypotheses, test statistic with degrees of freedom, *p*-value, and a brief conclusion in words). What is the relationship between these results and the confidence intervals?
- 5. Plot the residuals versus the *predicted* salary and *each* of the explanatory variables (i.e., 4 residual plots). Are there any unusual patterns?
- 6. Examine the assumption of normality for the residuals using a qqplot and histogram. State your conclusions.
- 7. Predict the salary for a mathematician with quality index equal to 5.2, 15 years of experience, and publication index equal to 6.5 . Provide a 85% prediction interval with your prediction.