Homework 0A (Review?) due? Jan. 10

The following problem set is based from information in Appendix A. This is review or new material depending on which statistics course you took as a pre-requisite.

1. (From A.3) A certain automobile manufacturer equips a particular model with either a six-cylinder engine or a four-cylinder engine. Let X_1 and X_2 be fuel efficient for independently and randomly selected six-cylinder and four-cylinder cars, respectively. The means and standard deviations are as follows:

$$\begin{array}{ll} \mathsf{E}\{\mathsf{X}_1\} = 22 & \sigma\{\mathsf{X}_1\} = 1.2 \\ \mathsf{E}\{\mathsf{X}_2\} = 26 & \sigma\{\mathsf{X}_2\} = 1.5 \end{array}$$

a) Calculate the mean and standard deviation of $D = X_1 - X_2$.

b) Calculate the mean and standard deviation of $T = X_1 + X_2$.

c) Calculate the mean and standard deviation of A = T/2.

2. (A.4) Suppose the tensile strength of type-A steel has a distribution of $\mathcal{M}(\mu = 105 \text{ ksi}, \sigma^2 = 64 \text{ ksi}^2)$ and the tensile strength of type-B steel has a distribution of $\mathcal{M}(\mu = 100 \text{ ksi}, \sigma^2 = 36 \text{ ksi}^2)$. Assume that the strengths of the two types of steel are independent. Let T = A + B.

a) Describe the probability distribution of T.

- b) Find the probability that the total strength of these two types of steel exceeds 200 ksi?
- c) Find the probability that the total strength is between 190 and 200 ksi?

d) Obtain the 10th percentile of the probability distribution of T. Interpret this quantity.

3. (A.3) Suppose that the pH of a certain chemical compound is 5.00 and the pH measured by a randomly selected beginning chemistry student is 5.00 with variance 0.04.

- a) For a random sample of 55 students, what is the probability that the sample mean will be within ± 0.01 of the population mean?
- b) Would the probability be increased by 20 percent if the sample size were increased by 20 percent, to n = 66? Be specific, but do not perform any calculations.

4. (A.6) It is important in a safe workplace that workers are not asked to perform tasks, such as lifting, that exceed their capabilities. The following data is for the maximum weight of lift (MAWL, kg) for five randomly selected healthy males with ages from 18 – 30.

25.8 36.6 26.3 21.8 27.2Let μ be the mean MAWL weight for healthy males in this age range. Assume that MAWL are normally distributed.

a) Test the hypothesis that H_0 : $\mu \le 22$ versus H_a : $\mu > 22$ at a significance level of 0.05.

b) Construct a 99% confidence interval for μ .

5. (A.7, A.9) The maximum lean angle (the furthest a subject is able to lean and still recover in one step) for a random sample of 10 younger females (21 – 29 years) has an average of $\mu_Y = 30.7$ and a standard deviation of $\sigma_Y = 2.751$ and the data for a random sample of 5 older females (67 – 81 years) has an average of $\mu_O = 16.2$ and a standard deviation of $\sigma_O = 4.438$. Assume that the two sets of observations constitute independent random samples from normal populations.

a) Test the hypothesis that H₀: $\sigma_Y^2 = \sigma_0^2$ versus H_a: $\sigma_Y^2 \neq \sigma_0^2$ at a significance level of 0.10.

b) Assuming a common variance, calculate a 90% confidence interval for $\mu_{Y} - \mu_{O}$.