Review for Exam 2

Tables provided: normal distribution, t distribution (from Website)

Chapter 3

- 1. Negative binomial distribution (Note: work does NOT include the formula for the combination)
 - a) Be able to determine if a given situation follows the conditions for a negative binomial distribution note: X is the number of failures
 - b) Be able to calculate the probability of rth success among n trials (pmf):

$$nb(x;r,p) = P(X = x) = {\binom{x+r-1}{r-1}}p^r(1-p)^x, x = 0,1,2,...$$

c) Calculate the mean, variance and standard deviation of a negative binomial distribution

$$E(X) = \frac{r(1-p)}{p}, Var(X) = \frac{r(1-p)}{p^2}$$

- 2. Geometric distribution
 - a) Be able to determine if a given situation follows the conditions for a geometric distribution Note: Y = total number of trials.
 - b) Be able to calculate the probability of the first success in a series of trials (pmf): $g(y; p) = P(Y = y) = (1 - p)^{y-1}p, y = 1, 2, ...$
 - c) Calculate the mean, variance and standard deviation of a geometric distribution

$$E(Y) = \frac{1}{p}, Var(Y) = \frac{1-p}{p^2}$$

- d) All geometric distributions may be calculated using the negative binomial assuming that the random variable is defined appropriately.
- 3. Poisson distribution
 - a) Be able to calculate the probability of x successes (pmf):

$$p(x; \lambda) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

- b) Calculate the mean, variance and standard deviation of a Poisson distribution $E(X) = Var(X) = \lambda$
- c) Be able to determine if the Poisson distribution may be used to approximate the binomial distribution
- d) For the Poisson process, be able to calculate the probability

$$P(X = k) = \frac{e^{-\lambda t} (\lambda t)^k}{k!}$$

Chapter 4 (for continuous functions)

- 4. Be able to determine if a pdf is legitimate and be able to determine a correct pdf for any given functional form.
- 5. Be able to calculate a probability from a probability density function (pdf).
- 6. Be able to calculate a cdf from a pdf and vice versa.
- 7. Be able to calculate probabilities from a cdf.
- 8. Be able to calculate a percentile from either a pdf or a cdf.

- 9. Be able to calculate the expected value (mean), expected value of a function, variance and standard deviation given a pdf.
- 10. For the uniform (continuous) distribution:

a) Be able to use the pdf and cdf to calculate probabilities and percentiles

$$f(x; A, B) = \begin{cases} \frac{1}{B - A} & A \le x < B \\ 0 & else \end{cases}, \quad F(x) = \begin{cases} 0 & x < A \\ \frac{x - A}{B - A} & A \le x < B \\ 1 & x \ge B \end{cases}$$

b) Be able to calculate E(X) and Var(X).

$$E(X) = \frac{A+B}{2}, Var(X) = \frac{(B-A)^2}{12}$$

- 11. For the normal distribution:
 - a) Be able to state the applications of the normal distribution.
 - b) Be able to use the z-table to calculate probabilities and percentiles of any normal distribution.
 - c) Be able to approximate an appropriate Binomial distribution using the continuity correction.
 - i) valid if $np \ge 10$ and $nq \ge 10$
 - ii) Continuity correction:

$$\begin{array}{lll} & \text{Original} & \text{Approximation} \\ P(X = a) & P(a - 0.5 < X < a + 0.5) \\ P(a < X) & P(a + 0.5 < X) \\ P(a \le X) & P(a - 0.5 < X) \\ P(X < b) & P(X < b - 0.5) \\ P(X \le b) & P(X < b + 0.5) \end{array}$$

- 12. For the exponential distribution:
 - a) Be able to use the pdf and cdf to calculate probabilities and percentiles.

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0\\ 0 & else \end{cases}, \quad F(x;\lambda) = \begin{cases} 0 & x < 0\\ 1 - e^{-\lambda x} & x \ge 0 \end{cases}, \quad P(X > x) = \begin{cases} 0 & x < 0\\ e^{-\lambda x} & x \ge 0 \end{cases}$$

b) Be able to calculate E(X) and Var(X).
$$E(X) = \frac{1}{\lambda}, \text{Var}(X) = \frac{1}{\lambda^2} \end{cases}$$

13. For a normal probability plot (QQ plot), be able to determine if the distribution is normal, symmetrical but not normal, or skewed.

Chapter 5

- 14. Given a joint pmf or a joint pdf
 - a) Be able to calculate probabilities.
 - b) Be able to calculate the marginal pmf or pdf, respectively.
 - c) Determine if the r.v.'s are independent
- 15. If the two r.v.'s are independent, be able to calculate the joint pmf or pdf from the marginal pmf or pdf.
- 16. Given a joint pmf or joint pdf and the appropriate marginal pmf or marginal pdf, calculate the conditional pmf or pdf.
- 17. Be able to calculate probabilities, means and variances from a conditional pmf or a conditional pdf.
- 18. Be able to calculate the expected values, covariance and correlation from a joint pmf or joint pdf.

- 19. Be able to apply the properties of covariance and correlation.
- 20. Multinomial distribution
 - a) Be able to determine if a given situation follows the conditions for a multinomial distribution: the same as for binomial except for there are more than two possibilities
 - b) Be able to calculate the probability for a specific number of successes for each of the possibilities (pmf):

$$p(x_1, \dots, x_r) = \begin{cases} \frac{n!}{x_1! \cdots x_r!} p_1^{x_1} \cdots p_r^{x_r} & x_i = 0, 1, 2, \dots \text{ with } x_1 + \dots + x_r = n \\ 0 & else \end{cases}$$

- 21. Given a normal distribution, be able to calculate the expectation, variance and probabilities of the average and the sample total.
- 22. Using the Central Limit Theorem, be able to determine if the average or sample total from an unknown underlying distribution can be considered normal with appropriate mean and variance.
- 23. Be able to calculate the mean and variance of linear combination of r.v.. Note: the equation for variance simplifies if the r.v. are independent.
- 24. Be able to calculate probabilities from a linear combination of normal r.v.'s.

Chapter 6 (MC only)

- 25. Be able to define a point estimate and a point estimator.
- 26. Be able to state what an unbiased or biased point estimator is.
- 27. Be able to state which estimator for the mean and variance is unbiased for all distributions and which estimator is the MVUE for the normal distribution.
- 28. Be able to state why the MVUE is so important.
- 29. Given the graphs of two estimators, be able to determine which one is better.

Chapter 7

- 30. Be able to interpret the meaning of a confidence interval. This includes what is random about the interval.
- 31. Be able to determine what affects the width of the confidence interval and what the effect is.
- 32. Be able to state the assumptions that are required for the each of the following cases:
 - a) mean, Case I: normal distribution with known $\boldsymbol{\sigma}$
 - b) mean, Case III: normal (approximately normal) with unknown $\boldsymbol{\sigma}$
 - c) population proportion

33. Be able to calculate and interpret the CI in the following cases (this includes which box to use):

$$\mu \quad Case \ l: \overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \qquad Case \ III: \overline{x} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}} \\ p \quad \frac{\hat{p} + \frac{z_{\alpha/2}^2}{2n} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z_{\alpha/2}^2}{4n^2}}}{1 + \frac{z_{\alpha/2}^2}{n}} \qquad \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ \end{array}$$

*: This equation is not needed for the exam, but useful for the homework.

If the question says 'interpret', this means that the conclusion needs to be stated in words as per what was stated in class: "We are xx confident that true average of [situation in words] is in the interval [give the interval].

34. Be able to calculate the lower and upper bound for each of the cases in Objective 33.

35. Be able to calculate the sample size, n, needed for a particular width, w:

μ	$n = \left(2z_{\alpha/2} \cdot \frac{\sigma}{w}\right)^2$	$n = \left(2t_{\alpha/2, n-1}^* \cdot \frac{s}{w}\right)^{2^{\ddagger}}$
р	$n pprox rac{4 z_{lpha/2}^2 \hat{p} \hat{q}^\dagger}{w^2}$	

†: p̂ may be known or not known.

‡: t* uses the value of n that was provided in the preliminary study. The calculated n needs to be large than the n in the preliminary study for this method to be valid. I will not accept a trial and error method as correct on the exam.