Examples for Counting Techniques

1) Roll two 8-sided dice. What is the probability that the sum of the two numbers is 5? (0.0625)

N(**A**) = 4
Possibilities (1,4), (2,3), (3,2), (4,1)
N = 8^2 (product rule)
$P(A) = \frac{N(A)}{N} = \frac{4}{64} = 0.0625$

2) Draw two cards from a suit of 13 cards (say diamonds), what is the probability that the sum of the two cards is even? (A = 1, J = 11, Q = 12, K = 13)? (0.462)

N(A): unordered without replacement, the sum is even if both are odd or both are even
$N(A) = number\ for\ both\ even + number\ for\ both\ odd = \binom{7}{2} + \binom{6}{2} = \frac{7!}{5!\,2!} + \frac{6!}{4!\,2!} = 21 + 15 = 36$
N: unordered without replacement, all possibilities of drawing 2 cards from 13
$N = \binom{13}{2} = \frac{13!}{11!\,2!} = 78$
$P(A) = \frac{N(A)}{N} = \frac{36}{78} = 0.462$

3. The IRS decides that it will audit the returns of 3 people from a group of 18. If 8 of the people are women, what is the probability that all 3 of people audited are women?

$N(A) = \binom{8}{3} = \frac{8!}{(8-3)!\,3!} = 56$
$N = \binom{18}{3} = \frac{18!}{(18-3)!\,3!} = 816$
$P(A) = \frac{N(A)}{N} = \frac{56}{816} = 0.0686$

4. Arizona plates consist of three digits followed by three letters. What is the probability that a particular license plate doesn't have any repeating digits or letters?

N(A) = $P_{3,10}$ $P_{3,26}$ = (10)(9)(8)(26)(25)(24) = 11,232,000
N = $10^3 26^3$ = 17,576,000
$P(A) = \frac{11{,}232{,}000}{17{,}576{,}000} = 0.639$