

STAT 350 Exam 2: Practice Exam

Spring 2016

Name (Print) : _____

PUID _____

Instructor (circle one): Findsen Monter Tooman Womble

Class Time (circle one): 9:30 am 11:30 AM 12:30 PM

1:30 PM 2:30 PM 3:30 PM ONLINE

Instructions:

1. You are expected to uphold the honor code of Purdue University. It is your responsibility to keep your work covered at all times. Anyone cheating on the exam will automatically fail the course and will be reported to the Office of Dean of Students.
2. Please let us know if you observe or hear of any cheating on the exam. We highly appreciate it and will reward your efforts with one bonus point per instance.
3. It is strictly prohibited to smuggle this exam outside. Your exam will be returned to you (online: you may pick up your exam from your instructor) after it is graded.
4. You may have one double-sided 8.5 in x 11 in crib sheet to take this test. The crib sheet can be handwritten or typed.
5. The only materials that you are allowed during the exam are your calculator, writing utensils, erasers and your crib sheet. If you bring any other papers into the exam, you will get a **zero** on the exam. Scratch paper will be provided if you need more room.
6. Leave all your belongings except those permitted for the exam in the front of the room. This includes your cell phone. We are not responsible for any loss.
7. If you share your calculator or use a cell phone, you will get a **zero** on the exam.
8. Breaks (including bathroom breaks) during the exam are **not** allowed. If you leave the exam room, you must turn in your exam and you will not be allowed to come back.
9. You must show **ALL** your work to obtain full credit. An answer without showing any work may result in **zero** credit.
10. All numeric answers should have **two decimal places** except answers from the z-table which should have **four decimal places**.
11. If your work is not readable, it will be marked wrong.
12. After you complete the exam, please turn in your exam as well as your crib sheet, tables and any scrap paper that you used. Please put your name on all pages that are stapled to the exam. Please be prepared to show your Purdue picture ID.

Your exam is not valid without your signature below.

I attest here that I have read and followed the instructions above honestly while taking this exam and that the work submitted is my own, produced without assistance from books, other people (including other students in this class), notes other than my own crib sheets, or other aids. In addition, I agree that if I tell any other student in this class anything about the exam BEFORE they take it, I (and the student that I communicate the information to) will fail the course and be reported to the Office of the Dean of Students for Academic Dishonesty.

Signature of Student: _____

	Points Earned	Grader
Name/Section/Signature (1 point)		
Problem 1 (Multiple Choice) (39 points)		
Problem 2 (32 points)		
Problem 3 (25 points)		
Problem 4 (20 points)		
Problem 5 (40 points)		
Total (105 / 100)		

Note: There 156 points on the exam, however, the midterm will be out of 105/100.

1. (39 points, 3 points each) Multiple Choice Questions (circle only one answer). If you do not circle the answer, you will receive 0 points for the question. There is only one correct answer per question.

- 1.1 A random sample of 35 koalas was obtained and each was carefully weighed. The average weight was 20.75 pounds and the sample standard deviation was 3.05. A 99% confidence interval for the population mean weight of all koalas is (19.422, 22.078). Determine which of the following statements is true:
- A. Researches are 99% confident that the CI captures the population mean.
 - B. Researches are 99% confident that the population mean lies in the CI.
 - C. Researches are 99% confident that the CI captures the sample mean.
 - D. Researches are 99% confident that the sample mean lies in the CI.
- 1.2 Determine which of the following statements is true concerning a hypothesis test:
- A. If $p\text{-value} > \alpha$ we have strong evidence to accept H_0
 - B. The $p\text{-value}$ is the probability that H_0 is true.
 - C. Small $p\text{-values}$ are evidence against H_0
 - D. Large $p\text{-values}$ give convincing evidence against H_0
- 1.3 The average time in years to get an undergraduate degree in computer science was compared for men and women. Random samples of 100 male computer science majors and 100 female computer science majors were taken. Choose the appropriate parameter(s) for this situation.
- A. One population mean μ_1
 - B. Difference between two population means $\mu_1 - \mu_2$ where they are independent.
 - C. Difference between two population means $\mu_1 - \mu_2$ where they are paired.
 - D. None of the above.
- 1.4 Null and alternative hypotheses are statements about:
- A. Population parameters.
 - B. Sample parameters.
 - C. Sample statistics.
 - D. It depends - sometimes population parameters and sometimes sample statistics.
- 1.5 A pitcher in the MLB wants to know the mean speed of his pitch for the last season. The head coach tells him that the 90% confidence upper bound for his mean pitching speed is 99.23 mph. He wishes to test $H_0: \mu = 105$ mph versus $H_a: \mu < 105$ mph at the 1% significance level. Determine which of the following statements is true.
- A. The 99% confidence upper bound is less than the 90% confidence upper bound.
 - B. We reject H_0 since the value 105 falls out of the 90% confidence upper bound and would therefore also fall out of the 99% confidence upper bound.
 - C. We fail to reject because the 90% confidence upper bound is smaller than 105.
 - D. We cannot make a decision since the confidence level we used to calculate the confidence upper bound is 90%, and we would need a 99% confidence upper bound.

- 1.6 A good way of determining the proper alternative hypothesis for a hypothesis test is to evaluate
- A. what parameter value(s) the experimenter assumes to be true without proof.
 - B. what the experimenter is trying to prove or detect about the parameter(s) being tested.
 - C. what the experimenter claims to be a valid value for the parameter being tested.
 - D. the likely consequence should H_0 be proven false.
- 1.7 To optimize the Atlanta bus service, the average number of minutes that busses in each of 20 randomly selected bus routes is late (compared to the published time) was recorded. You may assume that the times follow a normal distribution with an unknown mean μ and unknown standard deviation σ . Which of the following would produce a confidence interval with a smaller half-width than a 95% confidence interval?
- A. Compute a 99% confidence interval rather than a 95% confidence interval so that you are more confident of the result.
 - B. Record 10 bus routes rather than 20, because 10 routes are easier to measure.
 - C. Select bus routes that do not go through downtown to decrease variability.
 - D. Select bus routes to those routes that are less than 10 miles so that the average time of the route is shorter.
- 1.8. To be sure that a computer supply store is refilling their generic replacement ink cartridges to 30 mL of ink, a consumer group randomly sampled 17 black replacement cartridges. The results of the test stated that the ink was less than 30 mL to a 0.01 level of significance. Suppose that in reality, the amount of ink in the cartridges was 30 mL or more. Which of the following statements is **TRUE**?
- A. A Type I error has been committed.
 - B. A Type II error has been committed.
 - C. No error has been committed.
- 1.9. Which of the following is **NOT** true about the standard error of a statistic?
- A. The standard error measures, roughly, the average difference between the statistic and the population parameter.
 - B. The standard error is the estimated standard deviation of the sampling distribution for the statistic.
 - C. The standard error can never be a negative number.
 - D. The standard error increases as the sample size(s) increases.
- A or D, either is correct.
- 1.10A professor wishes to record information on some of his upper division students for later use. He randomly chooses five juniors and five seniors from his class. What best describes his sampling technique?
- A. SRS
 - B. stratified random sample design
 - C. matched pair design
 - D. block design
- 1.11 Which of the following should **NOT** be considered when designing an experiment:
- A. replication
 - B. randomization
 - C. control
 - D. size of the population

- 1.12 A behavioral scientist wishes to know more about the walking habits of university students across a particular campus sidewalk. She positions herself out of sight of the students who are walking and records the time required for several of them to pass between two fixed points along the sidewalk. Which of the following best describes this study?
- A. anecdote
 - B. observational study
 - C. experiment with a control
 - D. experiment with no control
- 1.13 A study was conducted to compare five different training programs for improving endurance. Forty subjects were randomly divided into five groups of eight subjects in each group. A different training program was assigned to each group. After 2 months, the improvement in endurance was recorded for each subject. A one-way ANOVA is used to compare the five training programs, and the resulting *p-value* is 0.014. At a significance level of 0.05, what is the appropriate conclusion about mean improvement in endurance?
- A. The average amount of improvement appears to be the same for all five training programs.
 - B. It appears that at least one of the five training programs has a different average amount of improvement.
 - C. The average amount of improvement appears to be different for each of the five training programs.
 - D. One training program is significantly better than the other four.

(32 pts.) 2. The director of a High School is concerned about how much time students spent on homework. Last year, the average time students spent on homework was 21 hours per week. This year, a random sample of 32 students indicates that the average time spent is 22.4 hours per week. Assume that the population standard deviation is 2.7 hours.

a. (5 pts.) What assumptions are required for this test to be valid?

(2 pts.) 1. SRS

(3 pts.) 2. the population or sampling distribution is normally distributed

3. (Optional) students sampled are independent of each other.

b. (5 pts.) Is this a z or a t test? Please explain your answer.

(3 pts.) Since σ is known,

(2 pts.) this is a z test.

c. (10 pts.) Perform the 4 step hypothesis test to determine whether there is any evidence to suggest that students spent more time on homework this year than last year. Please use a significance level of 1%.

(1 pts.) 1. μ = the population (or true) mean (or average) of hours per week spent on homework (by students this year – optional).

(1 pts.) 2. $H_0: \mu = 21$ $H_a: \mu > 21$

3.

$$(2 \text{ pts.}) z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} (\text{optional}) = \frac{22.4 - 21}{2.7/\sqrt{32}} = \frac{1.4}{0.4773} = 2.93$$

$$(2 \text{ pts.}) p\text{-value} = P(Z > 2.93) = 1 - P(Z < 2.93) = 1 - 0.9983 = 0.0017$$

4.

(1 pts.) Reject H_0 since $0.0017 < 0.01$

(3 pts.) The data does provide evidence ($P = 0.0017$) to the claim that the population mean hours per week spent on homework by students this year is greater than last year (or greater than 21).

d. (7 pts.) Construct a 99% lower confidence bound for the population mean. No interpretation is required

$$\begin{aligned} \mu > \bar{x} - (1 \text{ pt.}) z_{\alpha} \frac{\sigma}{\sqrt{n}} (1 \text{ pt.}) &= 22.4 - 2.3263(1 \text{ pt.}) \left(\frac{2.7}{\sqrt{32}} \right) (1 \text{ pt.}) = 22.4 - 1.11 \\ &= 21.29 (1 \text{ pt.}) \end{aligned}$$

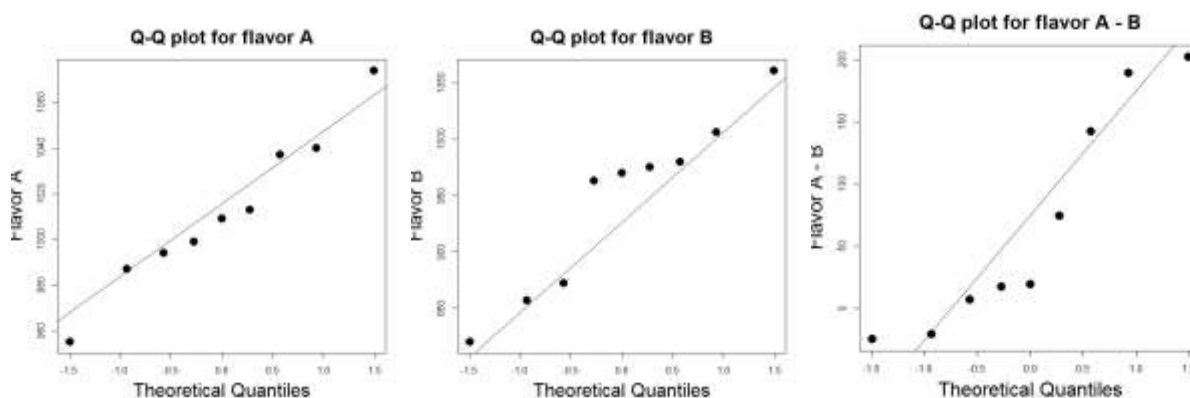
e. (5 pts.) Explain why parts c) and d) are consistent with each other.

The confidence bound states that we are 99% confidence that we are greater than 21.29 which is greater than 21 so it should be a reject H_0 .

(25 pts.) 3. An experiment was run to determine whether or not different flavors of ice cream melt at different speeds. Two flavors, A and B, of ice cream were stored in the same freezer in similar-sized containers. For each observation, one teaspoonful of ice cream was taken from the freezer, transferred to a plate, and the melting time at room temperature was observed to the nearest second. Nine observations were taken on each flavor. These are shown in the following table, and the normal probability plots of these data are shown below.

Flavor	Time in Seconds									Mean	SD
A	1009	987	955	1074	994	1040	1037	999	1013	1012	34.7
B	820	970	980	872	975	1061	963	857	1006	944.9	78.2
A - B	189	17	-25	202	19	-21	74	142	7	67.1	89.0

The calculated degrees of freedom using the Satterthwaite approximation is 11.03.



- a. (5 points) Should you use the two sample independent or two-sample pairs procedure to analyze the data? Explain your answer. You will receive a **0** points if the reason relates to the data that is provided in the question.

(3 pts) We should use two-sample independent procedure,

(2 pts) Explain: There is no way to pair the variables or there are no confounding variables.

- b. (5 pts.) Using the graphs provided, are the assumptions met to perform the inference? Please explain your answer. Be sure to indicate which graphs you are using.

For both situations, we have to assume that the samples (pairs) are independent. In addition, there is a normality assumption. For the QQ plots, it is normal if the points are close to the line.

Look at the paragraph depending on what is stated in a).

Two-Sample Independent

(2 pts.) You would look at graphs for Flavor A and Flavor B.

(3 pts.) For Flavor A, the points are close to the line. For Flavor B, the points in the middle are a little off, but with a sample size of $9 + 9 = 18$, this should be good enough. Accept full credit if they say that this is not acceptable though as long as the explanation is that the points are not close enough to the line.

Two-Sample Paired

(2 pts.) You would look at the graph for flavor A - B.

(3 pts.) The points are close to the line.

- c. (10 points) No matter what your answer is in b), construct and interpret a 95% confidence interval for the mean difference in melting time for the two flavors. Remember, we will grade on consistency from part a).

Independent

$$(2 \text{ pt.}) (\bar{x}_A - \bar{x}_B) \pm t_{\alpha/2, v} \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}$$

critical value using $df = 11$, $t_{\alpha/2, v} = t_{0.025, 11}$ (1 pt.) = 2.2010 (1 pt.)

$$(1012 - 944.9) \pm 2.2010 \sqrt{\frac{34.7^2}{9} + \frac{78.2^2}{9}} (1 \text{ pt.}) = 67.1 \pm 2.2010 \sqrt{813.26} = 67.1 \pm 2.2010(28.52)$$

$$= 67.1 \pm 62.77$$

(1 pt.) (4.33, 129.87)

- (3 pts) We are 95% confident (1 pt.) that the true or population (0.5 pts.) mean (0.5 pts.) Different of melting for the two flavors (0.5 pts.) is between 4.33 seconds and 129.87 seconds or is in the interval (4.33, 129.87) (0.5 pts.).

Paired

$$\bar{d} \pm (1 \text{ pt.}) t_{\alpha/2, n-1} \frac{s_D}{\sqrt{n}} (1 \text{ pt.}) = 67.1 \pm t_{0.025, 8} (1 \text{ pt.}) \frac{89.0}{\sqrt{9}} (1 \text{ pt.}) = 67.1 \pm 2.3060 (1 \text{ pt.}) \frac{89.0}{3}$$

$$= 67.1 \pm 68.41 \Rightarrow (-1.31, 135.51) (1 \text{ pt.})$$

- (3 pts) We are 95% confident (1 pt.) that the true or population (0.5 pts.) mean (0.5 pts.) different of melting for the two flavors (0.5 pts.) is between -1.31 seconds and 135.51 seconds or is in the interval (-1.31, 135.51) (0.5 pts.).

- d. (5 points) Please write an English sentence stating whether you think that these two flavors of ice cream melt in the same amount of time. To help in your decision, it is not possible to determine differences of melting times of less than 10 seconds

independent

(3 pts.) No because 0 is not in the interval.

(5 pts.) Even though 0 is not in the interval, 4.33 is within 10 of 0 so you really cannot make a decision on whether they are different or not.

paired

(3 pts.) Yes because 0 is in the interval.

(5 pts.) Even though 0 is in the interval, -1.31 is within 10 of 0 so you really cannot make a decision on whether they are different or not.

(20 points) 4. A cosmetics manufacturer of a hairspray product states that their product has a hold time of 24 hours. To test this claim, we randomly sample 30 people who use the hairspray and obtained a sample mean hold time 18.4 hours with a sample standard deviation of 5.4 hours.

- a. (10 pts.) Construct a 95% confidence interval for the length of time that the hairspray holds. Be sure to interpret your interval.

$$\begin{aligned}\bar{x} \pm (1 \text{ pt.}) t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} (1 \text{ pt.}) &= 18.4 \pm t_{.025, 29} (1 \text{ pt.}) \frac{5.4}{\sqrt{30}} (1 \text{ pt.}) \\ &= 18.4 \pm 2.0452 (1 \text{ pt.}) \frac{5.4}{\sqrt{30}} = 18.4 \pm 2.016 \Rightarrow (16.384, 20.416)\end{aligned}$$

We are 95% confident (1 pt.) that the true or population (0.5 pts.) mean (0.5 pts.) of the hold time of this hairspray product (0.5 pts.) is between 16.384 and 20.416 hours or is in the interval (16.384, 20.416) (0.5 pts.)

- b. (5 pts.) Using the preliminary information provided in the question, what approximate sample size is required so that the half-width of the confidence interval is at most 1.5 hours with a confidence level of 95%?

$$\text{Half-width of the confidence interval } ME = t_{.025, 29} \frac{s}{\sqrt{n}} (1 \text{ pt.}) = 2.0452 \frac{5.4}{\sqrt{n}} = \frac{11.043}{\sqrt{n}}.$$

Thus, we need

$$\frac{11.043}{\sqrt{n}} \leq 1.5 \Rightarrow 11.043 \leq 1.5\sqrt{n} \Rightarrow \frac{11.043}{1.5} \leq \sqrt{n} \Rightarrow 7.362 \leq \sqrt{n} \Rightarrow (7.36)^2 \leq n \Rightarrow 54.17 \leq n$$

Since n must be an integer, $n \geq 55$ (1 pt. round up)

OR

$$\begin{aligned}(1 \text{ pt.}) n &= \left(\frac{t_{.025, 29} s}{ME} \right)^2 \\ (3 \text{ pts.}) &= \left(\frac{(2.0452)(5.4)}{1.5} \right)^2 = 7.36^2 = 54.17 \\ (1 \text{ pts}) \text{ round up } &\rightarrow n = 55\end{aligned}$$

- c. (5 pts.) Without performing the hypothesis test, is the manufacture's claim correct at a 5% significance level? Explain your answer. You will receive **0 points** if your explanation includes the calculation of the test statistic or the P value.

Since 24 is not included in our 95% confidence interval (2 pts.), we can reject the null hypothesis (3 pts.) that the true population mean of the hold time of this hairspray product is 24 hours.

(40 points) 5. The weight gain of women during pregnancy has an important effect on the birth weight of their children. If the weight gain is not adequate, the infant is more likely to be small and will tend to be less healthy. In a study conducted in three countries, weight gains (in kilograms) of women during the third trimester of pregnancy were measured. The results are summarized in the following table:

Country	N	Mean	StDev
Egypt	46	3.7	2.5
Kenya	100	3.1	1.8
Mexico	52	2.9	1.8

Analysis of Variance				
Source	SS	MS	F-Value	P-Value
Country(Model)	17.22	8.61	xxx	0.1321
Error	767.25	xxxx		

- a. (5 pts.) Is it reasonable to use the assumption of equal standard deviation when analyzing these data? Give a reason for your answer.

1pts Yes
 4pts $\frac{2.5}{1.8} = 1.39 < 2$

- b. (5 pts.) Find the value of MSE. What is the standard deviation?

1pts DFE = 46+100+52 - 3 = 195
 2pts MSE = SSE / DFE = 767.25/195 = **3.93**
 2pts $s_p = \sqrt{3.93} = 1.98$ (This was not covered in this class)

- c. (5 pts.) What are the numerator and denominator degrees of freedom for the F statistic?

2pts DF2 (Denominator) = DFE = 195
 3pts DF1 (Numerator) = 3 - 1 = 2

- d. (5 pts.) Find the value of the F statistic.

2pts F = MSM/MSE
 3pts F = 8.61/3.93 = **2.19**

Note: An alternative way to ask parts b, c and d is to ask you to fill in the ANOVA table.

- e. (10 pts.) Carry out a significance test to compare the mean birth weights for the three countries at a significance level 0.05. Please follow the four step procedure and make a conclusion in context.

Step 1	2 pt.	Let μ_1 is the population mean weight gains (in kilograms) of women during the third trimester of pregnancy in Egypt Let μ_2 is " in Kenya Let μ_3 is " in Mexico
Step 2	1pt	$H_0: \mu_1 = \mu_2 = \mu_3$
	1pts	H_a : not all of the μ_i are equal
Step 3	1pt	$F(2, 195) = 2.19$ (Note the work was done above)
	1pts	P-Value = 0.1321
Step 4	2pts	Fail to reject H_0 because $0.132 > 0.05$
	2pts	The data does not show support ($P = 0.132$) to the claim that the population mean weight gains of women during the third trimester of pregnancy in the three countries are different.

- f. (5 pts.) Should a multiple comparison test be performed in this situation? Please explain your answer in one sentence.

2pts	No
3pts	We concluded that the population weight gains of women during the third trimester of pregnancy in the three countries are the same so no further analysis is needed.

- g. (5 pts.) No matter what your answer is in the previous question, what would be the critical value and Standard Error using the Tukey multiple comparison method? If the conclusion in part e) was reject, the following question would be asked: What would be the smallest weight gain during pregnancy? Please explain your answer. A listing of the confidence intervals would be included.

3 pts.

$$\text{critical value} = \frac{Q_{0.05,3,195}}{\sqrt{2}} = \frac{Q_{0.05,3,120}}{\sqrt{2}} = \frac{3.356}{\sqrt{2}} = 2.37$$

2 pts. any of the SE's will be given full credit

$$SE = \sqrt{MSE \left(\frac{1}{n_i} + \frac{1}{n_j} \right)} = \sqrt{3.93 \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}$$

$$SE_{\text{Egypt-Kenya}} = \sqrt{3.93 \left(\frac{1}{46} + \frac{1}{100} \right)} = 0.35$$

$$SE_{\text{Egypt-Mexico}} = \sqrt{3.93 \left(\frac{1}{46} + \frac{1}{52} \right)} = 0.40$$

$$SE_{\text{Kenya-Mexico}} = \sqrt{3.93 \left(\frac{1}{100} + \frac{1}{52} \right)} = 0.34$$