

## STAT 350 Practice Exam

Spring 2016

Name (Print) : \_\_\_\_\_

PUID \_\_\_\_\_

Instructor (circle one):      Findsen      Monter      Tooman      Womble

Class Time (circle one):      9:30 am      11:30 AM      12:30 PM

1:30 PM      2:30 PM      3:30 PM      ONLINE

**Instructions:**

1. You are expected to uphold the honor code of Purdue University. It is your responsibility to keep your work covered at all times. Anyone cheating on the exam will automatically fail the course and will be reported to the Office of Dean of Students.
2. Please let us know if you observe or hear of any cheating on the exam. We highly appreciate it and will reward your efforts with one bonus point per instance.
3. It is strictly prohibited to smuggle this exam outside. Your exam will be returned to you (online: you may pick up your exam from your instructor) after it is graded.
4. You may have one double-sided 8.5 in x 11 in crib sheet to take this test. The crib sheet can be handwritten or typed.
5. The only materials that you are allowed during the exam are your calculator, writing utensils, erasers and your crib sheet. If you bring any other papers into the exam, you will get a **zero** on the exam. Scratch paper will be provided if you need more room.
6. Leave all your belongings except those permitted for the exam in the front of the room. This includes your cell phone. We are not responsible for any loss.
7. If you share your calculator or use a cell phone, you will get a **zero** on the exam.
8. Breaks (including bathroom breaks) during the exam are **not** allowed. If you leave the exam room, you must turn in your exam and you will not be allowed to come back.
9. You must show **ALL** your work to obtain full credit. An answer without showing any work may result in **zero** credit.
10. All numeric answers should have **two decimal places** except answers from the z-table which should have **four decimal places**.
11. If your work is not readable, it will be marked wrong.
12. After you complete the exam, please turn in your exam as well as your crib sheet, tables and any scrap paper that you used. Please be prepared to show your Purdue picture ID.

**Your exam is not valid without your signature below.**

I attest here that I have read and followed the instructions above honestly while taking this exam and that the work submitted is my own, produced without assistance from books, other people (including other students in this class), notes other than my own crib sheets, or other aids. In addition, I agree that if I tell any other student in this class anything about the exam BEFORE they take it, I (and the student that I communicate the information to) will fail the course and be reported to the Office of the Dean of Students for Academic Dishonesty.

Signature of Student: \_\_\_\_\_

	Points Earned	Grader
Name/Section (1 point)		
Problem 1 (Multiple Choice) (15 points)		
Problem 2 (10 points)		
Problem 3 (15 points)		
Problem 4 (25 points)		
Problem 5 (30 points)		
Total (105 / 100)		

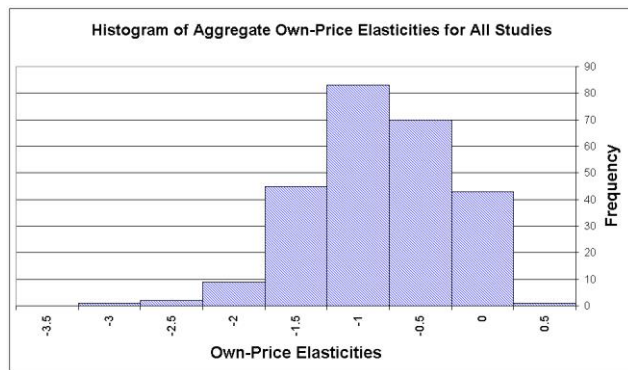
Note: There are only 95 points on the exam, however, the midterm will be out of 105.

1. (15 points, 3 points each) Multiple Choice Questions (circle only one answer). If you do not circle the answer, you will receive 0 points for the question. There is only one correct answer per question.

1.1 Inferential statistics may be best defined as:

- ☒ A. techniques and methods used to analyze a small, specific set of data in order to draw a conclusion about a large, more general collection of data.
- ☐ B. graphical and numerical methods used to describe, organize, and summarize a statistically valid conclusion.
- ☐ C. graphical and numerical methods used to describe, organize, and summarize data.
- ☐ D. techniques and methods used to analyze a small data set to make inference concerning its accuracy.

1.2 The following histogram shows aggregate own-price elasticities for 21 different studies ([http://www.fin.gc.ca/consultresp/airtravel/airtravstdy\\_2-eng.asp](http://www.fin.gc.ca/consultresp/airtravel/airtravstdy_2-eng.asp)). Which of the following describes the shape of the histogram?



- ☒ A. negatively skewed
- ☐ B. symmetric
- ☐ C. positively skewed
- ☐ D. None of the above.

1.3 In which of the following situations is the Central Limit Theorem not applicable?

- ☐ A. When the sample is small and the population is normal
- ☐ B. When the sample is large and the population is normal
- ☐ C. When the sample is large, above 30, and the population is not normal
- ☒ D. When the sample is small, below 30, and the population is not normal

1.4 Researchers were interested in whether uniform colors give athletes an advantage over their competitors. To investigate this, they considered 457 matches where there were two colors of the competitors, red or blue. They looked at the number of matches until the first competitor with a blue uniform won. This random variable can be modeled as

- ☐ A. Binomial
- ☐ B. Poisson
- ☐ C. Exponential
- ☒ D. none of the above.

1.5 In the normal distribution, the normal curve becomes wider and flatter because of a

- ☐ A. smaller value of mean.
- ☒ B. larger value of standard deviation.
- ☐ C. smaller value of standard deviation.
- ☐ D. larger value of mean.

**(10 pts.) 2.** Consider a uniform random variable  $X$ , with probability density function:

$$f_X(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the expected value of  $X^2$ .

$$E[X] = \int_{-\infty}^{\infty} x^2 f(x) dx$$

by definition

$$E[X] = \int_0^2 x^2 \left(\frac{1}{2}\right) dx$$

4 pts, 1 pt. limits, 1 pt. 'x<sup>2</sup>', 1 pt. f(x), 1 pt. 'dx'

$$= \frac{1}{2} \int_0^2 x^2 dx$$

$$= \frac{1}{2} \left[ \frac{x^3}{3} \right]_0^2$$

3 pts. (or correct integration of what they have)

$$= \frac{1}{2} \left[ \frac{2^3}{3} - \frac{0^3}{3} \right]$$

$$= \frac{1}{2} \left[ \frac{8}{3} \right]$$

2 pts. work

$$= \frac{4}{3} = 1.33$$

1 pt.

If they just write down the answer after the integral, that is -5.

**(15 points) 3.** The following data shows the waiting times for 11 elective eye surgeries in number of days.

5	8	12	12	14	14	16	16	20	25	33
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↑

↑ (1 pt.)  
 $\bar{x}$ 

↑

- a. (5 points) What are the median,  $Q_1$  and  $Q_3$  for this data? You may show some of your work above.

work: 1 pt for arrow for median, 1 pt. for calculating  $d_1$  and  $d_3$

(1 pt.) Median =  $\bar{x} = 14$

$$d_1 = \frac{11}{4} = 2.75 \implies 3 \implies (1 \text{ pt.}) Q_1 = 12$$

$$d_3 = \frac{3(11)}{4} = 8.25 \implies 9 \implies (1 \text{ pt.}) Q_3 = 20$$

- b. (8 points) Are there any outliers in this data? If there are any outliers, please state what they are. Justify your answer.

(2 pts.)  $IQR = Q_3 - Q_1 = 20 - 12 = 8$  (2 pts.)

$$1.5 * IQR = 12$$

(2 pts.)  $IF_L$ : Low outliers: anything below  $Q_1 - 1.5 * IQR = 12 - 12 = 0 \implies$  (1 pt.) no outlier

(2 pts.)  $IF_H$ : High outliers: anything above  $Q_3 + 1.5 * IQR = 20 + 12 = 32$

(1 pt.) Yes, there is one outlier, 33.

- c. (2 points) What is the five number summary for this data. No work is required.

Each part is worth 0.5 pts

Minimum = 5

$Q_1 = 12$

median = 14

$Q_3 = 20$

maximum = 33

**(25 pts.) 4.** Betty rolls a fair 8-sided die 5 times. Hint: what is this distribution?

- a. (10 pts.) What is the probability that she rolls a "3" more than once?

(3 pts. – 1 for using binomial, 1 for the correct value of n and p). Note if they do not use a binomial distribution, they will lose 3 pts, then grade on consistency)

This is a binomial distribution with  $n = 5$  trials and probability of success,  $p = \frac{1}{8} = 0.125$ .

$$\begin{aligned} (2 \text{ pts., do not take off points if not written}) P(X > 1) &= (2 \text{ pts.}) 1 - P(X=0) - P(X=1) \\ &= (2 \text{ pts. – correct formulas}) 1 - \binom{5}{0}(.125)^0(.875)^5 - \binom{5}{1}(.125)^1(.875)^4 \\ &= 1 - (1)(1)(.513) - (5)(.125)(.586) \\ &= (1 \text{ pt.}) 0.121 \end{aligned}$$

- b. (5 pts.) Given that she rolls a "3" more than once, what is the probability that she rolls the "3" two times?

$$\begin{aligned} (1 \text{ pt, do not take off points if not written}) P(X=2|X>1) &= \frac{P(X=2 \cap X>1)}{P(X>1)} \\ &= (2 \text{ pts., do not take off points if not written}) \frac{P(X=2)}{P(X>1)} \\ &= (1 \text{ pt. for formula consistent with above}) \frac{\binom{5}{2}(.125)^2(.875)^3}{0.121} \\ &= \frac{(10)(.016)(.670)}{0.121} = \frac{0.107}{0.121} = (1 \text{ pt.}) 0.884 \end{aligned}$$

- c. (5 pts.) What is the expected number of times that she rolls a "3"?

$$(2 \text{ pts. formula, do not take off points if not written}) E[X] = np = (2 \text{ pts.}) (5)(0.125) = (1 \text{ pt.}) 0.625$$

- d. (5 pts.) If the expected number of times that she rolls a "3" on another die Y is 0.8, what is  $E(2X - 3Y)$ ?

$$\begin{aligned} E[2X - 3Y] &= E[2X] + E[-3Y] \\ &= (2 \text{ pts.}) 2E[X] - 3E[Y] \\ &= (2 \text{ pts.}) 2(0.625) - 3(0.8) = 1.25 - 2.4 \\ &= (1 \text{ pt.}) -1.15 \end{aligned}$$

**(30 pts.) 5.** The amount of regular unleaded gasoline purchased every week at a gas station near UCLA follows a normal distribution with mean 50 thousand gallons and standard deviation 9 thousand gallons.

- a. (10 pts.) Find the probability that the amount of unleaded gasoline purchased will be above 60 thousand gallons.

$$\begin{aligned}
 (2 \text{ pts, partial credit only}) P(X > 60,000) &= (2 \text{ pts. for } z \text{ calculation}) P\left(Z > \frac{60,000 - 50,000}{9,000}\right) \\
 &= (1 \text{ pt.}) P(Z > 1.11) \\
 &= (2 \text{ pts. for the '1 -'}) 1 - P(Z < 1.11) \\
 &= (2 \text{ pts. for the value from the table}) 1 - 0.8665 = (1 \text{ pt.}) 0.1335
 \end{aligned}$$

- b. (10 pts.) How much unleaded gasoline is purchased if the amount is in the top 15%?

work: 3 pts, answer 2 pts.

$$P(Z > b) = 0.15 \implies 1 - P(Z < b) = 0.15 \implies P(Z < b) = 0.85 \implies b = 1.04$$

work: 4 pts., answer 1 pt.

$$\text{Then } b = \frac{x - 50,000}{9,000} \implies 1.04 = \frac{x - 50,000}{9,000} \implies 9,360 = x - 50,000 \implies (1 \text{ pt.}) 59,360 = x$$

That is, at least 59,360 gallons.

- c. (10 pts.) Find the probability that the AVERAGE amount of unleaded gasoline purchased in 6 weeks will be above 60 thousand gallons.

(1 pts.)  $\mu_{\bar{x}} = 50,000$  (the value in part a)

$$(1 \text{ pts.}) \sigma_{\bar{x}} = \frac{9000}{\sqrt{6}} = 3674.23$$

$$\begin{aligned}
 P(\bar{X} > 60,000) &= (2 \text{ pts for } z \text{ calculation}) P\left(Z > \frac{60,000 - 50,000}{9,000/\sqrt{6}}\right) = (1 \text{ pt.}) P(Z > 2.72) \\
 &= (2 \text{ pts. for the '1 -'}) 1 - P(Z < 2.72) \\
 &= (2 \text{ pts. for the value from the table}) 1 - 0.9967 = (1 \text{ pt.}) 0.0033
 \end{aligned}$$

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