My Five Friendly Facts about Independent Discrete Random Variables

1. Variance of a constant (not in the book)

Theorem: A random variable has a zero variance iff it is a constant random variable That is, Var(X) = 0 iff there is a constant c such that P(X = c) = 1

proof:

Show that if P(X = c) = 1, then Var(X) = 0. (Worksheet)
hint: use the definition of variance and Y = $(X - \mu_X)^2$

Var(X) = 0 $\rightarrow$ P(X = c) = 1

Let Y = $(X - \mu_X)^2$

0 = E(Y) = $\sum_y y p_Y(y) = \sum_{y \geq 0} y p_Y(y)$ [y is non-negative because it is a square]
Since $p_Y(y) \geq 0$ and $y \geq 0$, the only way that the sum can be 0 is if $p_Y(y) = 0$ if $y > 0$
Therefore, since $\sum_y y p_Y(y) = 1$, $p_y(0) = 1$, therefore P(Y = 0) = P(X = μ_X) = 1

2. Product

Theorem 12.17. The expected value of the product of the functions of two independent random variable equals the product of the expected values of the functions of the two random variables.
If X and Y are independent random variables, and g and h are any two functions then,
$$\mathbb{E}(g(X)h(Y)) = \mathbb{E}(g(X))\, \mathbb{E}(h(Y))$$

3. Linearity of Variance

Corollary 12:20. If X is any random variable, and if a and b are any constants then
Var(aX + b) = a^2 Var(X)
(Proof on Worksheet. Hint: use the definition of variance and the rules for expected values.)
$$\sigma_{aX+b} = \sqrt{Var(aX+b)} = \sqrt{a^2 Var(X)} = |a|\sqrt{Var(X)}$$

4. Standardized Random Variable (not in the book)

Definition: Standardized Random Variable
Let X be a random variable with finite nonzero variance. Then the standardized random variable corresponding to X, denoted X*, is defined by
$$X^* = \frac{X - \mathbb{E}(X)}{\sqrt{Var(X)}} = \frac{X - \mu_X}{\sigma_X}$$
Note: The standardized random variable corresponding to a random variable X represents the number of standard deviations that X is from its mean. Consequently, this is unitless
Worksheet: Calculate E(X*) and Var(X*)

5. Variance of a sum

Theorem 12.19. If $X_1, \ldots, X_n$ are independent random variables, and $a_1, \ldots, a_n$ are constants, then

$$Var(a_1X_1 + \cdots + a_nX_n) = a_1^2 Var(X_1) + \cdots a_n^2 Var(X_n)$$

Note:
1) This is only valid if X and Y are independent.
 if X and Y are dependent, then the middle term(s) do not cancel so there more terms in the sum
2) Var (X – Y) = Var (X + Y)