SET DEFINITIONS

Item	Definition	Designation	example
set	collection of objects	{1,3, 5,7}	my deck of cards is a set of cards
Empty set	a set has nothing in it	$\emptyset$	
Subset	A is a subset of B, designated by $A \subset B$ or $B \supset A$ if all members of A are members of B	$U = \{x \in \mathbb{Z}: P(x)\}$ where P(x) means $1 \le x \le 7$, x is odd this uses subsets to define the sets	
Equal sets	Two sets are equal if they contain the same elements, i.e., $A \subset B$ and $B \subset A$		
Proper subset	$A \subset B$ and $B \not\subset A$		

Number designations

$\mathbb{R}$ or $\mathcal{R}$	collection of real numbers
$\mathbb{Q}$ or $\mathcal{Q}$	collection of rational numbers, that is consists of p/q where p and q are integers
$\mathbb{Z}$ or $\mathcal{Z}$	collection of integers: …, -2, -1, 0, 1, 2, ...
$\mathbb{N}$ or $\mathcal{N}$	collection of positive integer (does not include 0): 1, 2, …

Intervals

Let $a,b \in \mathbb{R}$

$(a,b) = \{x \in \mathbb{R}: a < x < b\}$	bounded open interval	$[a,b] = \{x \in \mathbb{R}: a \le x \le b\}$	bounded closed interval
$[a,b) = \{x \in \mathbb{R}: a \le x < b\}$	bounded half-open interval	$(a,b] = \{x \in \mathbb{R}: a < x \le b\}$	bounded half-open interval
$(a,\infty) = \{x \in \mathbb{R}: x > a\}$	unbounded open interval	$(-\infty,b) = \{x \in \mathbb{R}: x < b\}$	unbounded open interval
$[a,\infty) = \{x \in \mathbb{R}: x \ge a\}$	bounded open interval	$(-\infty,b] = \{x \in \mathbb{R}: x \le b\}$	bounded open interval

Definition 1.2

Let U be a set and let A, B, and E be subsets of U

a) The **complement** of E, $E^C = \{x: x \notin E\}$

b) The **intersection** of A and B is $A \cap B = \{x: x \in A \text{ and } x \in B\}$

c) The **union** of A and B is $A \cup B = \{x: x \in A \text{ or } x \in B\}$

Proposition 1.1: De Morgan's Laws

Let A and B be subsets of U, Then

a) $(A \cup B)^c = A^c \cap B^c$ b) $(A \cap B)^c = A^c \cup B^c$

Proposition 1.2. Distributive Laws

Let A, B, and C be subsets of U, Then

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proposition 1.3: Associative and Commutative Laws

Let A, B, and C be subsets of U, Then

a) $A \cap B = B \cap A$ b) $A \cup B = B \cup A$

c) $A \cap (B \cap C) = (A \cap B) \cap C$ d) $A \cup (B \cup C) = (A \cup B) \cup C$

Definition 1.3
Let U be a set
a) for a finite collection, $A_1, A_2, \ldots, A_N$ of subsets of U

Intersection

$$\bigcap_{n=1}^{N} A_n = \{x : x \in A_n \text{ for all } n = 1,2,\ldots,N\}$$

Union

$$\bigcup_{n=1}^{N} A_n = \{x : x \in A_n \text{ for some } n = 1,2,\ldots,N\}$$

b) if we have a countably infinite collection $A_1, A_2, \ldots$ of sets, we have

Intersection

$$\bigcap_{n=1}^{\infty} A_n = \{x : x \in A_n \text{ for all } n = 1,2,\ldots,N\}$$

Union

$$\bigcup_{n=1}^{\infty} A_n = \{x : x \in A_n \text{ for some } n = 1,2,\ldots,N\}$$

Proposition 1.4 de Morgan's Laws
Let $A_1, A_2, \ldots$ be subsets of U, Then

$$a)\ \left(\bigcap_n A_n\right)^c = \bigcup_n A_n^c \qquad b)\ \left(\bigcup_n A_n\right)^c = \bigcap_n A_n^c$$

Proposition 1.5. Distributive Laws
Let B and $A_1, A_2, \ldots$ be subsets of U, Then

$$a)\ B \cap \left(\bigcup_n A_n\right) = \bigcup_n (B \cap A_n) \qquad b)\ B \cup \left(\bigcap_n A_n\right) = \bigcap_n (B \cup A_n)$$

Disjoint sets
Definition 1.4 Two sets, A and B are said to be **disjoint** if $A \cap B = \emptyset$, that is, they have no elements in common. Sets $A_1, A_2, \ldots$ are said to be **pairwise disjoint** if $A_n \cap A_m = \emptyset$ when $m \neq n$.

Example:
Let $U = \mathbb{R}$

a) Are the sets $\mathbb{Z}$ and (2,3) disjoint?
Yes because (2,3) does not contain any integers.

b) Are the sets $\mathbb{Z}$ and [2,3) disjoint?
No because [2,3) contains the element 2 which is an integer.

Cartesian Product
Definition 1.5: Let A and B be two sets. The **Cartesian product** of A and B, denoted by A X B is the set of all ordered pairs (a,b), where $a \in A$ and $b \in B$. If $A_1, A_2, \ldots, A_n$ are sets, the **Cartesian product** is

$$\bigtimes_{n=1}^{N} A_n = \{(a_1, a_2, \ldots, a_N) : a_n \in A_N, 1 \le n \le N\}$$

Example:

Let A = {a,b} and B = {1,2}, then A X B = {(a,1),(a,2),(b,1),(b,2)}