Additional Notes for Negative Binomial Random Variables

Why is this called a negative binomial?

1) You can think of this as the 'opposite' or 'negative' of a binomial distribution.

Binomial	Negative Binomial
n fixed	X = the number of trials
X = number of successes	r = number of successes is fixed

2)

$$\binom{x-1}{r-1} = \binom{-r}{x-r}$$

Comparison of binomial and negative binomial random variables

	Binomial	Negative Binomial
Question	What is the probability that that you will roll 9 "1's in the first 40 rolls?	What is the probability that 40th roll will be the 9th '1'?
Distribution	X ~ binomial (n = 40, p = 0.05)	X ~ NegBinomial (r = 9, p = 0.05)
Meaning of X	X = # of rolls = 40	X = # of rolls until the 9th '1'
Probability	$P(X=9)=\binom{40}{9}0.05^9 0.95^{31} = 1.09\times 10^{-4}$	$P(X=40)=\binom{39}{8}0.05^9 0.95^{31} = 2.45\times 10^{-5}$

Number of failures

Just like for the geometric random variable, there is an alternative way of defining the negative binomial random variable using the number of failures.

Let Y = the number of failures until the rth success, then

$$p_Y(y) = \binom{r+y-1}{r-1} p^r (1-p)^y, x = 0,1,\ldots$$

This is equivalent to our definition because the number of trials is r + y, and the number of failures is y

Comparison of Expectations

Negative Binomial		Geometric	
X = number of trials	Y = number of failures	W = number of trials	Z = number of failures
$\mathbb{E}(X) = \frac{r}{p}$	$\mathbb{E}(Y) = \frac{r(1-p)}{p}$	$\mathbb{E}(W) = \frac{1}{p}$	$\mathbb{E}(Z) = \frac{1-p}{p}$

Comparison of Variances

Negative Binomial	Geometric
X = number of trials Y = number of failures	W = number of trials Z = number of failures
$Var(X) = Var(Y) = \frac{r(1-p)}{p^2}$	$Var(W) = Var(Z) = \frac{(1-p)}{p^2}$