Bayes' 5: Bayes Theorem and Tree Diagrams

There is another more intuitive way to perform Bayes' Theorem problems without using the formula. That is, using a Tree Diagram. If you look at how a tree diagram is created, these are really conditional probabilities.

If we want to determine a conditional probability, the formula is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

In Bayes' Theorem problem, we don't know P(A|B), however we do know P(B|A). I will illustrate how to do the problem by using Example 2 from the readings.

Example: Suppose that we have two dice in a hat (one has 6 sides and one has 20 sides). Pick one of the dice at random (each die is chosen with probability ½). If we obtain a "5" on the die when we roll it, what is the probability that the die had 20 sides?

Solution using a tree diagram:

As always, we will start by writing down all of the information.

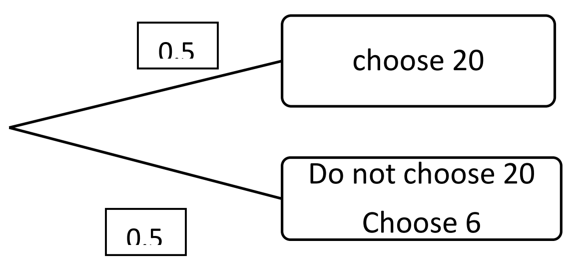
Definitions:
20 = choose the 20 sided die
5 = the value is a 5.

Given: P(20) = ½, $P(20^C)$ = ½, $P(5|20^C) = \frac{1}{6}$, $P(5|20) = \frac{1}{20}$
Want: P(20|5).

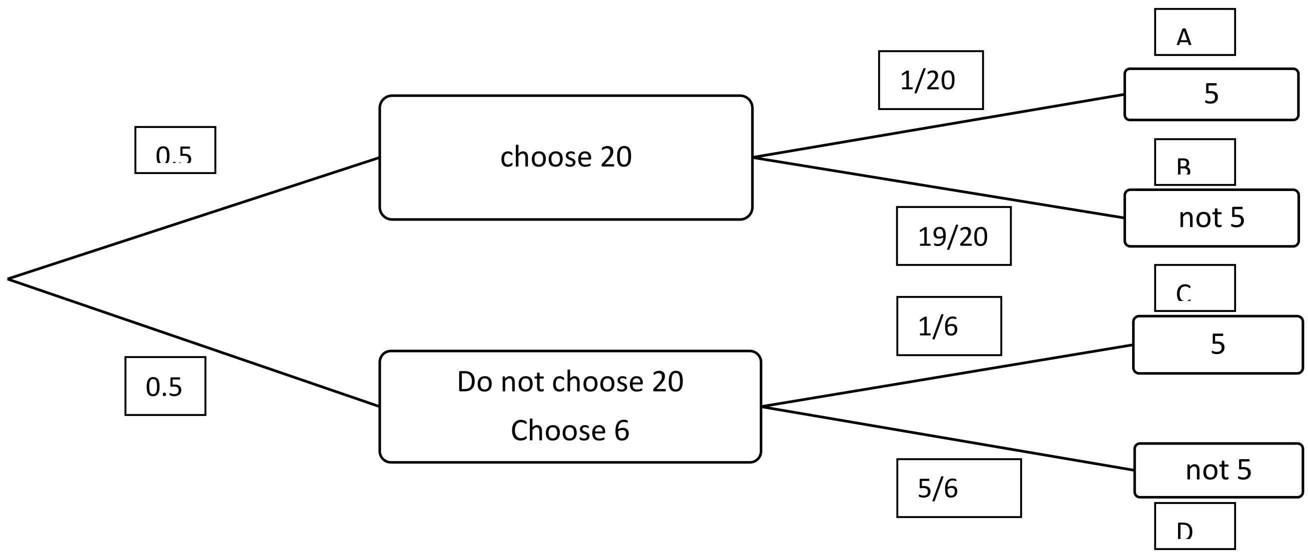
Note that this is a Bayes' Theorem problem because the conditional probability is 'backwards' of what is given.

In our tree diagram, what should go first? choosing a 20 sided die or obtaining a 5?
In what you are given, your conditions are based off of choosing a 20 sided die, so that is what should be first.

The tree diagram is:



Now we can add in our conditional probabilities.



We want to find P(20|5).

$$P(20|5) = \frac{P(20 \cap 5)}{P(5)}$$

What is P(20 ∩ 5)?

This is P(20)P(5|20) which is row A.

What P(5)?

This is going to be the sum of row A and row C where the 5 are.

Therefore:

$$P(20|5) = \frac{P(20 \cap 5)}{P(5)} = \frac{(0.5)\left(\frac{1}{20}\right)}{(0.5)\left(\frac{1}{20}\right) + (0.5)\left(\frac{1}{6}\right)} = \frac{3}{13} = 0.231$$