Review of Calculus

Derivatives:

Definition of Derivative

In geometric terms, the derivative is the slope of a curve at a particular point.

$$f'(x) = \frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

using an alternative definition, if x + h = c, then

$$f'(c) = \lim_{x \to c} \frac{f(c) - f(x)}{c - x}$$

Definition of a partial derivative

This occurs when we hold all but one of the independent variables of a function constant and is written by $\frac{\partial f(x,y,z)}{\partial x}$ or $f_x(x,y,z)$. Here x is allowed to change and y and z are considered constant.

Differentiable

A function, f(x,y) is differentiable at (x_o,y_o) if $f_x(x_o,y_o)$ and $f_y(x_o,y_o)$ exist plus other conditions that are related to being continuous. We call f differentiable if it is differentiable at every point in its domain.

Note:

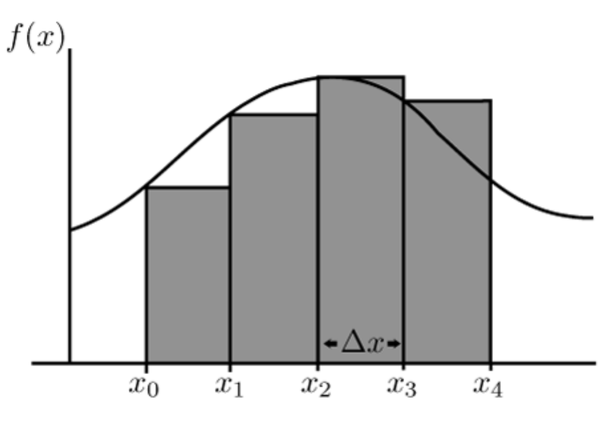
1) if the partial derivatives f_x and f_y of a function f(x,y) are continuous throughout an open region R, then f is differentiable at every point in R.
2) If a function f(x,y) is differentiable at (x_o,y_o) then f is continuous at (x_o,y_o)

Integrals:

Definition of an integral:

In geometric terms, the integral is the area under a curve.

Riemann Sum



To approximate the area using the Riemann sum, you add up all of the rectangles 'in' the curve to get the area. The diagram is a 'left-endpoint' sum. The height of each rectangle is $f(x_k)$ and the width is Δx_k. Therefore, the area can be approximated by:

$$S_p = \sum_{k=1}^{n} f(x_k)\Delta x_k$$

http://upload.wikimedia.org/wikipedia/commons/c/cc/Riemann_Sum_Left_Hand.png

Riemann Integral

This is the limit or the Riemann sum as Δx_k gets smaller and smaller and just equals the area under the curve.

$$\int_a^b f(x)dx = \lim_{\Delta x_k \to 0} \sum_{k=1}^{n} f(x_k)\Delta x_k$$

Fundamental Theorem of Calculus:

Let F(x) = $\int_a^x f(t)dt$, then F'(x) = f(x) and

$$\int_a^b f(x)dx = \int_a^b F'(x)dx = F(b) - F(a)$$

That is the integral of the derivative of a function can be evaluated by taking the antiderivative at the endpoints.

Another useful property of integrals is:

$$\int_{-\infty}^b f(x)dx - \int_{-\infty}^a f(x)dx = \int_a^b f(x)dx, \quad with\ a < b$$

You should be able to verify this by using the geometric definition of an integral.

Abbreviated table of integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int dx = \int x^0 dx = x + C \qquad \int e^{-ax}dx = -\frac{1}{a}e^{-ax} + C$$

$$\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$$ (This is done by substitution multiple times)

$$\int e^{-ax^2}dx = nothing\ in\ closed\ form \qquad \int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, a > 0$$

$$\int_{-\infty}^a e^{-ax^2}dx = nothing\ in\ closed\ form\ (see\ Table\ I, p. A-39\ in\ textbook)$$

$$\int xe^{-ax^2}dx = -\frac{1}{2a}e^{-ax^2}$$

Do this by substitution: let u = x^2 ==> du = 2xdx ==> xdx = $\frac{du}{2}$

$$\int xe^{-ax^2}dx = \frac{1}{2}\int e^{-au}du = -\frac{1}{2a}e^{-ax} = -\frac{1}{2a}e^{-ax^2}$$

We will mostly be using definite integrals so we use the Fundamental Theorem of Calculus to evaluate them. If there is an indefinite integral, then remember to always add in the 'C', the constant of integration.