EXAMPLE 5.6: *Injection Molded Sockets:*

Injection molding is used to make a particular socket, four pieces at a time. At the time these data were collected this method of fabrication represented a shift in both material and technology. Therefore, a process certification run was required prior to going into full production. Dave, the supervisor, decided to use control charts to evaluate this process certification run.

Since there was only one mold, the process certification run involved only one press, and only one operator. The data consisted of the effective thickness of a socket, measured in hundredths of a millimeter. Since one side of the socket was convex, a special gauge had to be designed and built to measure this thickness. Because of the way this gauge was built, the measurements represent the thickness in excess of 12.00 millimeters. Four times a day Dave would go to the press and gather up the parts produced by five consecutive cycles of the press. Since each cycle produced four parts, (one from each cavity) he had 20 parts to measure every two hours. Being careful, Dave kept track of the cycle and the cavity from which each part came. Having done this, he wrote his twenty measurements in the following array.

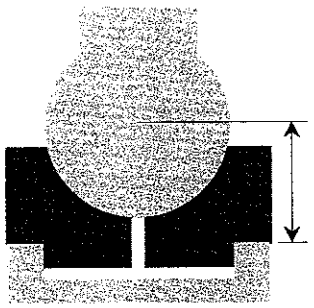

Figure 5.8: Socket Cross-Section

	Cycle of Press				
(Hour of Collection)	*A*	*B*	*C*	*D*	*E*
Cavity I	15	16	17	16	18
Cavity II	10	13	11	10	10
Cavity III	7	8	10	7	10
Cavity IV	8	9	10	10	10

Figure 5.9: Hourly Socket Thickness Data

There are three identifiable sources of variation within these data. There is the Hour-to-Hour variation that is represented by the different sets of 20 parts. There is the Cycle-to-Cycle variation that is represented by the different columns in Figure 5.9. And there is the Cavity-to-Cavity variation that is represented by the different rows in Figure 5.9.

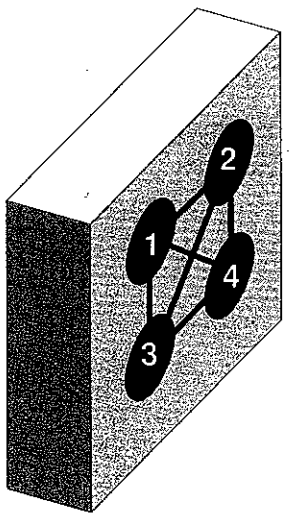

Figure 5.10: Socket Mold

The supervisor then had to choose how he would organize these data into subgroups, and how he would organize the subgroups into control charts. For purposes of illustration, this will be done in three different ways.

The first organization is shown in Table 5.2. Here the columns are used to form the subgroups. This organization allocates some sources of variation to be represented by differences between subgroups, and it allocates others to be represented by differences within the subgroups.

For example, the data for Hour 1 fall in different subgroups from that of Hour 2, therefore, the Hour-to-Hour differences will show up *between* the subgroups. Likewise, for any given hour, the data for Cycle A is in a different subgroup from that for Cycle B, etc., therefore the Cycle-to-Cycle differences will show up *between* the subgroups. Finally, for any given hour and cycle, the data for Cavity I falls within the same subgroup as that for Cavities II, III, and IV. Therefore, the Cavity-to-Cavity differences show up *within* the subgroups.

Table 5.2: First Organization of the Socket Data

1:	*A*	*B*	*C*	*D*	*E*
I	15	16	17	16	18
II	10	13	11	10	10
III	7	8	10	7	10
IV	8	9	10	10	10
$\bar{X}$	10	11.5	12	10.75	12
R	8	8	7	9	8

2:	*A*	*B*	*C*	*D*	*E*
I	13	18	15	15	15
II	9	10	11	8	9
III	7	11	10	10	9
IV	10	13	13	10	9
$\bar{X}$	9.75	13	12.25	10.75	10.5
R	6	8	5	7	6

3:	*A*	*B*	*C*	*D*	*E*
I	14	14	18	14	14
II	9	10	10	12	9
III	8	10	9	10	8
IV	8	9	10	8	9
$\bar{X}$	9.75	10.75	11.75	11	10
R	6	5	9	6	6

4:	*A*	*B*	*C*	*D*	*E*
I	14	15	15	15	14
II	10	10	11	11	10
III	7	9	12	10	10
IV	11	12	11	10	13
$\bar{X}$	10.5	11.5	12.25	11.5	11.75
R	7	6	4	5	4

5:	*A*	*B*	*C*	*D*	*E*
I	12	13	13	12	13
II	8	7	8	7	7
III	5	6	8	5	4
IV	4	4	5	3	4
$\bar{X}$	7.25	7.5	8.5	6.75	7
R	8	9	8	9	9

6:	*A*	*B*	*C*	*D*	*E*
I	14	15	17	14	13
II	7	11	12	8	6
III	6	4	7	6	5
IV	4	7	6	5	4
$\bar{X}$	7.75	9.25	10.5	8.25	7
R	10	11	11	9	9

7:	*A*	*B*	*C*	*D*	*E*
I	12	12	13	13	11
II	6	6	6	6	7
III	4	5	4	4	6
IV	4	4	5	4	5
$\bar{X}$	6.5	6.75	7	6.75	7.25
R	8	8	9	9	6

8:	*A*	*B*	*C*	*D*	*E*
I	13	15	16	14	13
II	9	8	7	7	8
III	10	8	6	7	5
IV	6	6	8	6	5
$\bar{X}$	9.5	9.25	9.25	8.5	7.75
R	7	9	10	8	8

9:	*A*	*B*	*C*	*D*	*E*
I	15	16	17	14	13
II	11	13	11	13	8
III	9	9	6	8	10
IV	9	10	11	9	5
$\bar{X}$	11	12	11.25	11	9
R	6	7	11	6	8

10:	*A*	*B*	*C*	*D*	*E*
I	13	13	14	13	13
II	7	8	7	6	7
III	4	5	6	5	5
IV	5	5	6	6	5
$\bar{X}$	7.25	7.75	8.25	7.5	7.5
R	9	8	8	8	8

11:	*A*	*B*	*C*	*D*	*E*
I	13	16	13	16	18
II	10	8	10	10	11
III	8	7	8	7	11
IV	11	10	9	10	10
$\bar{X}$	10.5	10.25	10	10.75	12.5
R	5	9	5	9	8

12:	*A*	*B*	*C*	*D*	*E*
I	16	18	18	16	13
II	13	8	8	10	7
III	13	10	10	6	7
IV	9	6	8	7	7
$\bar{X}$	12.75	10.5	11	9.75	8.5
R	7	12	10	10	6

13:	*A*	*B*	*C*	*D*	*E*
I	13	18	18	14	13
II	11	10	9	10	10
III	14	9	10	9	7
IV	9	12	11	11	6
$\bar{X}$	11.75	12.25	12	11	9
R	5	9	9	5	7

14:	*A*	*B*	*C*	*D*	*E*
I	13	13	14	14	14
II	7	8	9	8	8
III	6	6	7	6	6
IV	6	7	6	6	6
$\bar{X}$	8	8.5	9	8.5	8.5
R	7	7	8	8	8

15:	*A*	*B*	*C*	*D*	*E*
I	13	15	18	15	14
II	7	12	10	10	7
III	7	8	12	12	6
IV	11	12	12	12	6
$\bar{X}$	9.5	11.75	13	12.25	8.25
R	6	7	8	5	8

Table 5.2 Continued: First Organization of the Socket Data

16:	*A*	*B*	*C*	*D*	*E*
I	15	14	15	14	12
II	6	7	8	6	10
III	7	10	8	6	6
IV	6	8	7	9	7
$\bar{X}$	8.5	9.75	9.5	8.75	8.75
R	9	7	8	8	6

17:	*A*	*B*	*C*	*D*	*E*
I	14	16	14	13	14
II	9	10	13	7	6
III	6	5	5	7	5
IV	7	7	4	7	5
$\bar{X}$	9	9.5	9	8.5	7.5
R	8	11	10	6	9

18:	*A*	*B*	*C*	*D*	*E*
I	12	15	15	15	14
II	9	7	8	10	9
III	5	7	6	6	10
IV	6	8	7	12	12
$\bar{X}$	8	9.25	9	10.75	11.25
R	7	8	9	9	5

19:	*A*	*B*	*C*	*D*	*E*
I	17	15	18	18	17
II	10	10	12	12	6
III	5	9	7	8	7
IV	7	7	5	12	8
$\bar{X}$	9.75	10.25	10.5	12.5	9.5
R	12	8	13	10	11

20:	*A*	*B*	*C*	*D*	*E*
I	18	16	17	14	13
II	8	10	11	10	7
III	7	7	7	10	5
IV	10	7	9	8	8
$\bar{X}$	10.75	10	11	10.5	8.25
R	11	9	10	6	8

Thus, the subgroups defined in Table 5.2 allocate the major sources of variation in these data in the following manner:

Source of Variation	**First Allocation**
Hour to Hour	Between Subgroup
Cycle to Cycle	Between Subgroup
Cavity to Cavity	Within Subgroup

The Hour-to-Hour and Cycle-to-Cycle differences correspond to differences between subgroups while the Cavity-to-Cavity differences correspond to differences within the subgroups. One should not go on until this allocation is understood, for it is the key to understanding this example.

This first allocation yields 100 subgroups of size $n = 4$. The Grand Average is 9.74 units, and the Average Range is 7.90 units. The control limits for the Average Chart are 3.98 units to 15.50 units. The upper control limit for the Range Chart is 18.03 units. These control charts are shown in Figure 5.11.

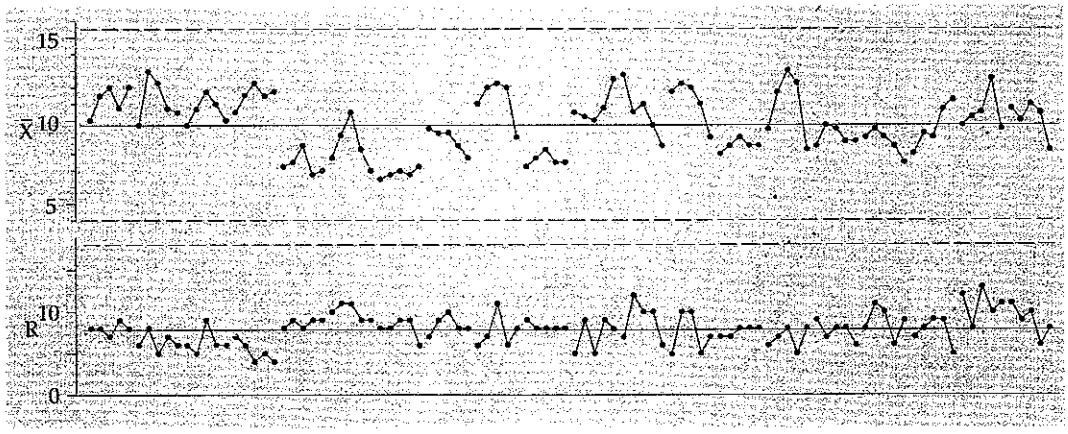

Figure 5.11: Control Charts for First Organization of Socket Data

Table 5.3: Second Organization of the Socket Data

1:	A	B	C	D	E	$\bar{X}$	R
I	15	16	17	16	18	16.4	3
II	10	13	11	10	10	10.8	3
III	7	8	10	7	10	8.4	3
IV	8	9	10	10	10	9.4	2

2:	A	B	C	D	E	$\bar{X}$	R
I	13	18	15	15	15	15.2	5
II	9	10	11	8	9	9.4	3
III	7	11	10	10	9	9.4	4
IV	10	13	13	10	9	11	4

3:	A	B	C	D	E	$\bar{X}$	R
I	14	14	18	14	14	14.8	4
II	9	10	10	12	9	10	3
III	8	10	9	10	8	9	2
IV	8	9	10	8	9	8.8	2

4:	A	B	C	D	E	$\bar{X}$	R
I	14	15	15	15	14	14.6	1
II	10	10	11	11	10	10.4	1
III	7	9	12	10	10	9.6	5
IV	11	12	11	10	13	11.4	3

5:	A	B	C	D	E	$\bar{X}$	R
I	12	13	13	12	13	12.6	1
II	8	7	8	7	7	7.4	1
III	5	6	8	5	4	5.6	4
IV	4	4	5	3	4	4	2

6:	A	B	C	D	E	$\bar{X}$	R
I	14	15	17	14	13	14.6	4
II	7	11	12	8	6	8.8	6
III	6	4	7	6	5	5.6	3
IV	4	7	6	5	4	5.2	3

7:	A	B	C	D	E	$\bar{X}$	R
I	12	12	13	13	11	12.2	2
II	6	6	6	6	7	6.2	1
III	4	5	4	4	6	4.6	2
IV	4	4	5	4	5	4.4	1

8:	A	B	C	D	E	$\bar{X}$	R
I	13	15	16	14	13	14.2	3
II	9	8	7	7	8	7.8	2
III	10	8	6	7	5	7.2	5
IV	6	6	8	6	5	6.2	3

9:	A	B	C	D	E	$\bar{X}$	R
I	15	16	17	14	13	15	4
II	11	13	11	13	8	11.2	5
III	9	9	6	8	10	8.4	4
IV	9	10	11	9	5	8.8	6

10:	A	B	C	D	E	$\bar{X}$	R
I	13	13	14	13	13	13.2	1
II	7	8	7	6	7	7	2
III	4	5	6	5	5	5	2
IV	5	5	6	6	5	5.4	1

11:	A	B	C	D	E	$\bar{X}$	R
I	13	16	13	16	18	15.2	5
II	10	8	10	10	11	9.8	3
III	8	7	8	7	11	8.2	4
IV	11	10	9	10	10	10	2

12:	A	B	C	D	E	$\bar{X}$	R
I	16	18	18	16	13	16.2	5
II	13	8	8	10	7	9.2	6
III	13	10	10	6	7	9.2	7
IV	9	6	8	7	7	7.4	3

13:	A	B	C	D	E	$\bar{X}$	R
I	13	18	18	14	13	15.2	5
II	11	10	9	10	10	10	2
III	14	9	10	9	7	9.8	7
IV	9	12	11	11	6	9.8	6

14:	A	B	C	D	E	$\bar{X}$	R
I	13	13	14	14	14	13.6	1
II	7	8	9	8	8	8	2
III	6	6	7	6	6	6.2	1
IV	6	7	6	6	6	6.2	1

15:	A	B	C	D	E	$\bar{X}$	R
I	13	15	18	15	14	15	5
II	7	12	10	10	7	9.2	5
III	7	8	12	12	6	9	6
IV	11	12	12	12	6	10.6	6

16:	A	B	C	D	E	$\bar{X}$	R
I	15	14	15	14	12	14	3
II	6	7	8	6	10	7.4	4
III	7	10	8	6	6	7.4	4
IV	6	8	7	9	7	7.4	3

17:	A	B	C	D	E	$\bar{X}$	R
I	14	16	14	13	14	14.2	3
II	9	10	13	7	6	9	7
III	6	5	5	7	5	5.6	2
IV	7	7	4	7	5	6	3

18:	A	B	C	D	E	$\bar{X}$	R
I	12	15	15	15	14	14.2	3
II	9	7	8	10	9	8.6	3
III	5	7	6	6	10	6.8	5
IV	6	8	7	12	12	9	6

19:	A	B	C	D	E	$\bar{X}$	R
I	17	15	18	18	17	17	3
II	10	10	12	12	6	10	6
III	5	9	7	8	7	7.2	4
IV	7	7	5	12	8	7.8	7

20:	A	B	C	D	E	$\bar{X}$	R
I	18	16	17	14	13	15.6	5
II	8	10	11	10	7	9.2	4
III	7	7	7	10	5	7.2	5
IV	10	7	9	8	8	8.4	3

The second organization of these data is shown in Table 5.3. Here the rows are used to define the subgroups and the allocation of the sources of variation is as follows: the Hour-to-Hour variation is represented by differences between subgroups; the Cycle-to-Cycle variation is represented by differences within the subgroups; and the Cavity-to-Cavity variation is represented by differences between subgroups.

Source of Variation	Second Allocation
Hour to Hour	Between Subgroup
Cycle to Cycle	Within Subgroup
Cavity to Cavity	Between Subgroup

This second allocation yields 80 subgroups of size $n = 5$. The Grand Average is unchanged at 9.74 units, and the Average Range is now 3.51 units. The control limits for the Average Chart are 7.71 to 11.77 units, and the upper control limit for the Range Chart is 7.42 units. The control charts for this arrangement of the data are shown in Figure 5.12.

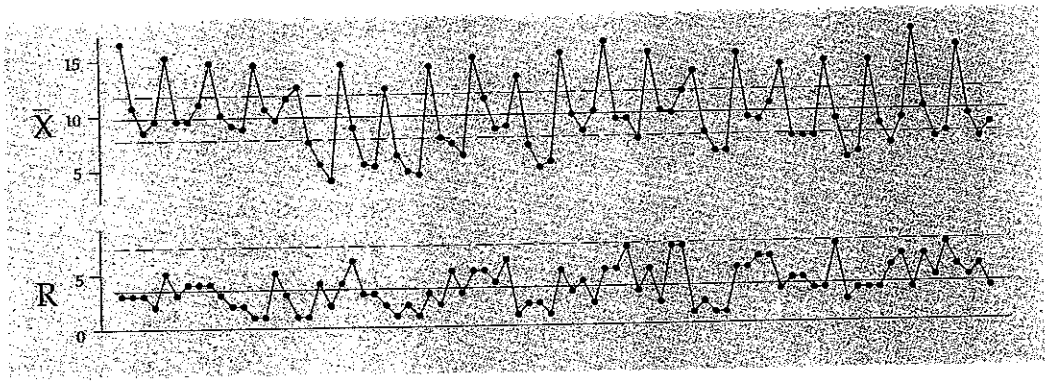

Figure 5.12: Control Charts for Second Organization of Socket Data

The control charts in Figures 5.11 and 5.12 are considerably different, yet they portray the same data. The difference is due to the way the data are organized into subgroups.

The Average Chart in Figure 5.12 shows an obvious pattern: every fourth average is out of control on the high side. Based on the order in which the subgroups were plotted, these high averages all correspond to Cavity I. Thus, Cavity I is producing sockets that are detectably thicker than those produced by the other three cavities.

Why isn't this difference seen in the charts in Figure 5.11? Because the right question wasn't asked. The allocation for the first control chart was:

Source of Variation	**First Allocation**
Hour to Hour	Between Subgroup
Cycle to Cycle	Between Subgroup
Cavity to Cavity	Within Subgroup

And this allocation results in charts which ask the following questions:

(1) Are there detectable differences from Hour-to-Hour?
(2) Are there detectable differences from Cycle-to-Cycle?
(3) Are the Cavity-to-Cavity differences consistent?

The charts in Figure 5.11 do **not** ask if there are detectable differences between the cavities.

In contrast, the control charts in Figure 5.12 have the following allocation:

Source of Variation	**Second Allocation**
Hour to Hour	Between Subgroup
Cycle to Cycle	Within Subgroup
Cavity to Cavity	Between Subgroup

And this allocation results in charts which ask the following questions:

(1) Are there detectable differences from Hour-to-Hour?
(2) Are the Cycle-to-Cycle differences consistent?
(3) Are there detectable differences from Cavity-to-Cavity?

Since the Cycle-to-Cycle differences are so much smaller than the Cavity-to-Cavity differences, the control limits in Figure 5.12 are much tighter than those in Figure 5.11. Thus, in addition to considering different questions, the charts in 5.12 are more sensitive.

The third organization of these data is simply a rearrangement of the subgroups in Table 5.3. The subgroups for each cavity were collected together and separate control charts were made for each of the four cavities.

Table 5.4: Third Organization of the Socket Data

Cavity I:

	A	B	C	D	E	$\bar{X}$	R
1:	15	16	17	16	18	*16.4*	3
2:	13	18	15	15	15	*15.2*	5
3:	14	14	18	14	14	*14.8*	4
4:	14	15	15	15	14	*14.6*	1
5:	12	13	13	12	13	*12.6*	1
6:	14	15	17	14	13	*14.6*	4
7:	12	12	13	13	11	*12.2*	2
8:	13	15	16	14	13	*14.2*	3
9:	15	16	17	14	13	*15*	4
10:	13	13	14	13	13	*13.2*	1
11:	13	16	13	16	18	*15.2*	5
12:	16	18	18	16	13	*16.2*	5
13:	13	18	18	14	13	*15.2*	5
14:	13	13	14	14	14	*13.6*	1
15:	13	15	18	15	14	*15*	5
16:	15	14	15	14	12	*14*	3
17:	14	16	14	13	14	*14.2*	3
18:	12	15	15	15	14	*14.2*	3
19:	17	15	18	18	17	*17*	3
20:	18	16	17	14	13	*15.6*	5

Cavity II:

	A	B	C	D	E	$\bar{X}$	R
1:	10	13	11	10	10	*10.8*	3
2:	9	10	11	8	9	*9.4*	3
3:	9	10	10	12	9	*10*	3
4:	10	10	11	11	10	*10.4*	1
5:	8	7	8	7	7	*7.4*	1
6:	7	11	12	8	6	*8.8*	6
7:	6	6	6	6	7	*6.2*	1
8:	9	8	7	7	8	*7.8*	2
9:	11	13	11	13	8	*11.2*	5
10:	7	8	7	6	7	*7*	2
11:	10	8	10	10	11	*9.8*	3
12:	13	8	8	10	7	*9.2*	6
13:	11	10	9	10	10	*10*	2
14:	7	8	9	8	8	*8*	2
15:	7	12	10	10	7	*9.2*	5
16:	6	7	8	6	10	*7.4*	4
17:	9	10	13	7	6	*9*	7
18:	9	7	8	10	9	*8.6*	3
19:	10	10	12	12	6	*10*	6
20:	8	10	11	10	7	*9.2*	4

Cavity III:

	A	B	C	D	E	$\bar{X}$	R
1:	7	8	10	7	10	*8.4*	3
2:	7	11	10	10	9	*9.4*	4
3:	8	10	9	10	8	*9*	2
4:	7	9	12	10	10	*9.6*	5
5:	5	6	8	5	4	*5.6*	4
6:	6	4	7	6	5	*5.6*	3
7:	4	5	4	4	6	*4.6*	2
8:	10	8	6	7	5	*7.2*	5
9:	9	9	6	8	10	*8.4*	4
10:	4	5	6	5	5	*5*	2
11:	8	7	8	7	11	*8.2*	4
12:	13	10	10	6	7	*9.2*	7
13:	14	9	10	9	7	*9.8*	7
14:	6	6	7	6	6	*6.2*	1
15:	7	8	12	12	6	*9*	6
16:	7	10	8	6	6	*7.4*	4
17:	6	5	5	7	5	*5.6*	2
18:	5	7	6	6	10	*6.8*	5
19:	5	9	7	8	7	*7.2*	4
20:	7	7	7	10	5	*7.2*	5

Cavity IV:

	A	B	C	D	E	$\bar{X}$	R
1:	8	9	10	10	10	*9.4*	2
2:	10	13	13	10	9	*11*	4
3:	8	9	10	8	9	*8.8*	2
4:	11	12	11	10	13	*11.4*	3
5:	4	4	5	3	4	*4*	2
6:	4	7	6	5	4	*5.2*	3
7:	4	4	5	4	5	*4.4*	1
8:	6	6	8	6	5	*6.2*	3
9:	9	10	11	9	5	*8.8*	6
10:	5	5	6	6	5	*5.4*	1
11:	11	10	9	10	10	*10*	2
12:	9	6	8	7	7	*7.4*	3
13:	9	12	11	11	6	*9.8*	6
14:	6	7	6	6	6	*6.2*	1
15:	11	12	12	12	6	*10.6*	6
16:	6	8	7	9	7	*7.4*	3
17:	7	7	4	7	5	*6*	3
18:	6	8	7	12	12	*9*	6
19:	7	7	5	12	8	*7.8*	7
20:	10	7	9	8	8	*8.4*	3

Instead of looking for differences between the cavities, this organization explicitly allows for such differences by placing different cavities on different charts.

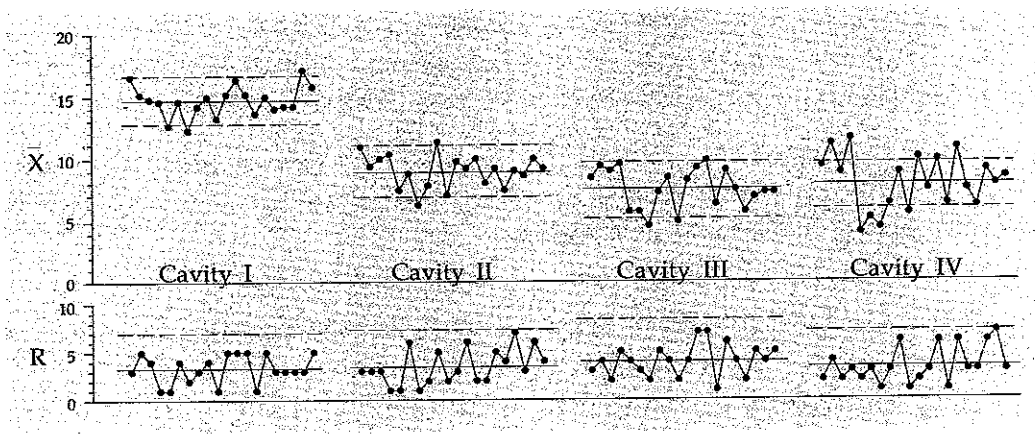

Figure 5.13: Control Charts for Third Organization of Socket Data

Cavity I has a Grand Average of 14.65, and an Average Range of 3.30, giving control limits for averages of 16.55 and 12.75, and an upper control limit for ranges of 6.98.

Cavity II has a Grand Average of 8.97, and an Average Range of 3.45, giving control limits for averages of 10.96 and 6.98, and an upper control limit for ranges of 7.29.

Cavity III has a Grand Average of 7.47, and an Average Range of 3.95, giving control limits for averages of 9.75 and 5.19, and an upper control limit for ranges of 8.35.

Cavity IV has a Grand Average of 7.86, and an Average Range of 3.35, giving control limits for averages of 9.79 and 5.93, and an upper control limit for ranges of 7.08.

By drawing the Average Charts on the same scale, the difference between Cavity One and the other three cavities becomes immediately apparent. Moreover, a lack of control is still indicated. With this organization, the long-term (Hour-to-Hour) variation is being directly compared with the short-term (Cycle-to-Cycle) variation. The charts show very clearly that the long-term variation is excessive. There is some Assignable Cause present that is making the press produce parts that are thinner one hour, and thicker the next hour. By identifying and removing this Assignable Cause, greater product consistency can be achieved with virtually no additional expense.

Of the three organizations, the third is the most sensitive, since it explicitly allows for the Cavity-to-Cavity variation. This allows the Hour-to-Hour lack of control to show up. This lack of control is present in all three charts, but it can be detected in the first chart, Figure 5.11, only by the use of Detection Rule III, and it is overwhelmed by the Cavity-