Measurement Uncertainty

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| **Background** A measurement result is complete only when accompanied by a quantitative statement of its uncertainty. The uncertainty is required in order to decide if the result is adequate for its intended purpose and to ascertain if it is consistent with other similar results. **International and U.S. perspectives on measurement uncertainty** Over the years, many different approaches to evaluating and expressing the uncertainty of measurement results have been used. Because of this lack of international agreement on the expression of uncertainty in measurement, in 1977 the International Committee for Weights and Measures (CIPM, *Comité International des Poids et Measures*), the world's highest authority in the field of measurement science (*i.e.*, metrology), asked the International Bureau of Weights and Measures (BIPM, *Bureau International des Poids et Mesures*), to address the problem in collaboration with the various national metrology institutes and to propose a specific recommendation for its solution. This led to the development of Recommendation INC-1 (1980) by the Working Group on the Statement of Uncertainties convened by the BIPM, a recommendation that the CIPM approved in 1981 and reaffirmed in 1986 via its own Recommendations 1 (CI-1981) and 1 (CI-1986):     Recommendation INC-1 (1980)Expression of experimental uncertainties 1. The uncertainty in the result of a measurement generally consists of several components which may be grouped into two categories according to the way in which their numerical value is estimated. Type A. Those which are evaluated by statistical methods Type B. Those which are evaluated by other means There is not always a simple correspondence between the classification into categories A or B and the previously used classification into "random" and "systematic" uncertainties. The term "systematic uncertainty" can be misleading and should be avoided. Any detailed report of uncertainty should consist of a complete list of the components, specifying for each the method used to obtain its numerical value.   |

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|  | 2. The components in category A are characterized by the estimated variances *si*2 ( or the estimated "standard deviations" *si*) and the number of degrees of freedom *vi*. Where appropriate the covariances should be given. 3. The components in category B should be characterized by quantities *uj*2, which may be considered approximations to the corresponding variances, the existence of which is assumed. The quantities *uj2* may be treated like variances and the quantities *uj* like standard deviations. Where appropriate, the covariances should be treated in a similar way. 4. The combined uncertainty should be characterized by the numerical value obtained by applying the usual method for the combination of variances. The combined uncertainty and its components should be expressed in the form of "standard deviations." 5. If for particular applications, it is necessary to multiply the combined uncertainty by an overall uncertainty, the multiplying factor must always be stated. The above recommendation, INC-1 (1980), is a brief outline rather than a detailed prescription. Consequently, the CIPM asked the International Organization for Standardization (ISO) to develop a detailed guide based on the recommendation because ISO could more easily reflect the requirements stemming from the broad interests of industry and commerce. The ISO Technical Advisory Group on Metrology (TAG 4) was given this responsibility. It in turn established Working group 3 and assigned it the following terms of reference: To develop a guidance document based upon the recommendation of the BIPM Working Group on the Statement of Uncertainties which provides rules on the expression of measurement uncertainty for use within standardization, calibration, laboratory accreditation, and metrology services; The purpose of such guidance is: to promote full information on how uncertainty statements are arrived at;to provide a basis for the international comparison

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| **International and U.S. perspectives*, continued*** The *Guide to the Expression of Uncertainty in Measurement*The end result of the work of ISO/TAG 4/WG 3 is the 100-page *Guide to the Expression of Uncertainty in Measurement* (or GUM as it is now often called). It was published in 1993 (corrected and reprinted in 1995) by ISO in the name of the seven international organizations that supported its development in ISO/TAG 4:

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| BIPM | Bureau International des Poids et Mesures |
| IEC | International Electrotechnical Commission |
| IFCC | International Federation of Clinical Chemistry |
| ISO | International Organization for Standardization |
| IUPAC | International Union of Pure and Applied Chemistry |
| IUPAP | International Union of Pure and Applied Physics |
| OIML | International Organization of Legal Metrology |

       The focus of the ISO *Guide* or GUM is the establishment of "general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields--from the shop floor to fundamental research." As a consequence, the principles of the GUM are intended to be applicable to a broad spectrum of measurements, including those required for:maintaining quality control and quality assurance in production; complying with and enforcing laws and regulations; conducting basic research, and applied research and development, in science and engineering; calibrating standards and instruments and performing tests throughout a national measurement system in order to achieve traceability to national standards; developing, maintaining, and comparing international and national physical reference standards, including reference materials.  |

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|  | Wide acceptance of the GUMThe GUM has found wide acceptance in the United States and other countries. For example: The GUM method of evaluating and expressing measurement uncertainty has been adopted widely by U.S. industry as well as companies abroad. The National Conference of Standards Laboratories (NCSL), which has some 1500 members, has prepared and widely distributed Recommended Practice RP-12, *Determining and Reporting Measurement Uncertainties*, based on the GUM. ISO published the French translation of the GUM in 1995, German and Chinese translations were also published in 1995, and an Italian translation was published in 1997. Translations of the GUM into Estonian, Hungarian, Italian, Japanese, Spanish, and Russian have been completed or are well underway. GUM methods have been adopted by various regional metrology and related organizations including:

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| NORAMET  | North American Collaboration in Measurement Standards |
| NAVLAP | National Voluntary Laboratory Accreditation Program |
| A2LA | American Association for Laboratory Accreditation |
| EUROMET | European Collaboration in Measurement Standards |
| EUROLAB | A focus for analytic chemistry in Europe |
| EA | European Cooperation for Accreditation |
| EU | European Union; adopted by CEN and published as EN 13005. |

       Moreover, the GUM has been adopted by NIST and most of NIST's sister national metrology institutes throughout the world, such as the National Research Council (NRC) in Canada, the National Physical Laboratory (NPL) in the United Kingdom, and the Physikalisch-Technische Bundesanstalt in Germany.        Most recently, the GUM has been adopted by the American National Standards Institute (ANSI) as an American National Standard. Its official designation is ANSI/NCSL Z540-2-1997 and its full title is *American National Standard for Expressing Uncertainty--U.S. Guide to the Expression of Uncertainty in Measurement*. This publication may be [ordered directly](http://physics.nist.gov/cuu/Uncertainty/isoorder.html) from NCSL.        It is noteworthy that NIST's adoption of the GUM approach to expressing measurement uncertainty was done with considerable forethought. Although quantitative statements of uncertainty had accompanied most NIST measurement results, there was never a uniform approach at NIST to the expression of uncertainty. Recognizing that the use of a single approach within NIST instead of a variety of approaches would simplify the interpretation of NIST outputs, and that U.S. industry was calling for a uniform method of expressing measurement uncertainty, in 1992 then NIST Director J. W. Lyons appointed a NIST Ad Hoc Committee on Uncertainty Statements to study the issue. In particular, the Ad Hoc committee was asked to ascertain if the GUM approach would meet the needs of NIST's customers. The conclusion was that it most definitely would, and a specific policy for the implementation of the GUM approach at NIST was subsequently adopted.        NIST Technical Note 1297 (TN 1297, online in a [pdf version](http://physics.nist.gov/Pubs/guidelines/TN1297/tn1297s.pdf) or in an [html version](http://physics.nist.gov/Pubs/guidelines/contents.html) -- see the [Bibliography](http://physics.nist.gov/cuu/Uncertainty/bibliography.html) for full citation) was prepared by two members of the Ad Hoc Committee, who also played major roles in the preparation of the GUM. (The policy, "Statement of Uncertainty Associated with Measurement Results," was incorporated in the NIST Administrative Manual and is included as Appendix C in TN 1297.) TN 1297 has in fact found broad acceptance. To date, over 40 000 copies have been distributed to NIST staff and in the United States at large and abroad -- to metrologists, scientists, engineers, statisticians, and others who are involved with measurement in some way. JCGM       Most recently, a new international organization has been formed to assume responsibility for the maintenance and revision of the GUM and its companion document the VIM (see the [Bibliography](http://physics.nist.gov/cuu/Uncertainty/bibliography.html) for a brief discussion of the VIM). The name of the organization is **Joint Committee for Guides in Metrology (JCGM)** and its members are the seven international organizations listed above: BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, and OIML, together with the International Laboratory Accreditation Cooperation (ILAC). ISO/TAG 4 has been reconstituted as the Joint ISO/IEC TAG, Metrology, and will focus on metrological issues internal to ISO and IEC as well as represent ISO and IEC on the JCGM. Further information regarding the JCGM may be found at [http://www.bipm.org/enus/2\_Committees/joint\_committees.html](http://www.nist.gov/cgi-bin/exit_nist.cgi?url=http://www.bipm.org/enus/2_Committees/joint_committees.html) (NOTE: Space in URL is actually **\_**).  |

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| **Essentials of expressing measurement uncertainty** This is a brief summary of the method of evaluating and expressing uncertainty in measurement adopted widely by U.S. industry, companies in other countries, NIST, its sister national metrology institutes throughout the world, and many organizations worldwide. These "essentials" are adapted from NIST Technical Note 1297 (TN 1297), prepared by B.N. Taylor and C.E. Kuyatt and entitled *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, which in turn is based on the comprehensive International Organization for Standardization (ISO) *Guide to the Expression of Uncertainty in Measurement*. Users requiring more detailed information may [access TN 1297 online](http://physics.nist.gov/Pubs/guidelines/TN1297/tn1297s.pdf), or if a comprehensive discussion is desired, they may [purchase the ISO *Guide*](http://physics.nist.gov/cuu/Uncertainty/isoorder.html). Background information on the development of the ISO *Guide*, its worldwide adoption, NIST TN 1297, and the NIST policy on expressing measurement uncertainty is given in the section [International and U.S. perspectives on measurement uncertainty](http://physics.nist.gov/cuu/Uncertainty/international1.html). To assist you in reading these guidelines, you may wish to consult a short [glossary](http://physics.nist.gov/cuu/Uncertainty/glossary.html). Additionally, a companion publication to the ISO *Guide,* entitled the *International Vocabulary of Basic and General Terms in Metrology*, or VIM, gives definitions of many other important terms relevant to the field of measurement. Users may also [purchase the VIM](http://physics.nist.gov/cuu/Uncertainty/isoorder.html). **Basic definitions** Measurement equation The case of interest is where the quantity *Y* being measured, called the **measurand**, is not measured directly, but is determined from *N* other quantities *X*1, *X*2, . . . , *XN* through a functional relation *f*, often called the **measurement equation**:

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| *Y* = *f*(*X*1, *X*2, . . . , *XN*) | (1) |

Included among the quantities *Xi* are corrections (or correction factors), as well as quantities that take into account other sources of variability, such as different observers, instruments, samples, laboratories, and times at which observations are made (e.g., different days). Thus, the function *f* of equation (1) should express not simply a physical law but a measurement process, and in particular, it should contain all quantities that can contribute a significant uncertainty to the measurement result. An estimate of the measurand or *output quantity* *Y*, denoted by *y*, is obtained from equation (1) using *input estimates* *x*1, *x*2, . . . , *xN* for the values of the *N* *input quantities* *X*1, *X*2, . . . , *XN*. Thus, the *output estimate* *y*, which is the result of the measurement, is given by

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| *y* = *f*(*x*1, *x*2, . . . , *xN*). | (2) |

For example, as pointed out in the ISO *Guide*, if a potential difference *V* is applied to the terminals of a temperature-dependent resistor that has a resistance *R*0 at the defined temperature *t*0 and a linear temperature coefficient of resistance *b*, the power *P* (the measurand) dissipated by the resistor at the temperature *t* depends on *V*, *R*0, *b*, and *t* according to

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| *P* = *f*(*V*, *R*0, *b*, *t*) = *V*2/*R*0[1 + *b*(*t* - *t*0)]. | (3) |

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|  | Classification of uncertainty components The uncertainty of the measurement result *y* arises from the uncertainties *u* (*xi*) (or *ui* for brevity) of the input estimates *xi* that enter equation (2). Thus, in the example of equation (3), the uncertainty of the estimated value of the power *P* arises from the uncertainties of the estimated values of the potential difference *V*, resistance *R*0, temperature coefficient of resistance *b*, and temperature *t*. In general, components of uncertainty may be categorized according to the **method** used to evaluate them. **Type A evaluation**method of evaluation of uncertainty by the **statistical analysis** of series of observations, **Type B evaluation**method of evaluation of uncertainty by means **other than the statistical analysis** of series of observations. Representation of uncertainty components **Standard Uncertainty**Each component of uncertainty, however evaluated, is represented by an estimated standard deviation, termed **standard uncertainty** with suggested symbol *ui*, and equal to the positive square root of the estimated variance **Standard uncertainty: Type A**An uncertainty component obtained by a Type A evaluation is represented by a statistically estimated standard deviation *si*, equal to the positive square root of the statistically estimated variance *si*2, and the associated number of degrees of freedom *vi*. For such a component the standard uncertainty is *ui = si*. **Standard uncertainty: Type B**In a similar manner, an uncertainty component obtained by a Type B evaluation is represented by a quantity *uj* , which may be considered an approximation to the corresponding standard deviation; it is equal to the positive square root of *uj*2, which may be considered an approximation to the corresponding variance and which is obtained from an assumed probability distribution based on all the available information. Since the quantity *uj*2 is treated like a variance and *uj* like a standard deviation, for such a component the standard uncertainty is simply *uj*.  |

**Evaluating uncertainty components:** Type A

A Type A evaluation of standard uncertainty may be based on any valid statistical method for treating data. Examples are calculating the standard deviation of the mean of a series of independent observations; using the method of least squares to fit a curve to data in order to estimate the parameters of the curve and their standard deviations; and carrying out an analysis of variance (ANOVA) in order to identify and quantify random effects in certain kinds of measurements.

Mean and standard deviation

As an example of a Type A evaluation, consider an input quantity *Xi* whose value is estimated from *n* independent observations *Xi ,k* of *Xi* obtained under the same conditions of measurement. In this case the input estimate *xi* is usually the **sample mean**

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and the standard uncertainty *u*(*xi*) to be associated with *xi* is the estimated **standard deviation of the mean**

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| **Evaluating uncertainty components:** Type BA Type B evaluation of standard uncertainty is usually based on scientific judgment using all of the relevant information available, which may include: * previous measurement data,
* experience with, or general knowledge of, the behavior and property of relevant materials and instruments,
* manufacturer's specifications,
* data provided in calibration and other reports, and
* uncertainties assigned to reference data taken from handbooks.

Below are some examples of Type B evaluations in different situations, depending on the available information and the assumptions of the experimenter. Broadly speaking, the uncertainty is either obtained from an outside source, or obtained from an assumed distribution.

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| Uncertainty obtained from an outside source |

**Multiple of a standard deviation** Procedure:  Convert an uncertainty quoted in a handbook, manufacturer's specification, calibration certificate, etc., that is a stated multiple of an estimated standard deviation to a standard uncertainty by dividing the quoted uncertainty by the multiplier.**Confidence interval** Procedure:  Convert an uncertainty quoted in a handbook, manufacturer's specification, calibration certificate, etc., that defines a "confidence interval" having a stated level of confidence, such as 95 % or 99 %, to a standard uncertainty by treating the quoted uncertainty as if a normal probability distribution had been used to calculate it (unless otherwise indicated) and dividing it by the appropriate factor for such a distribution. These factors are 1.960 and 2.576 for the two levels of confidence given. |

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| Uncertainty obtained from an assumed distribution |

**Normal distribution: "1 out of 2"** Procedure:  Model the input quantity in question by a normal probability distribution and estimate lower and upper limits *a*- and *a*+ such that the best estimated value of the input quantity is (*a*+ + *a*-)/2 (i.e., the center of the limits) and there is 1 chance out of 2 (i.e., a 50 % probability) that the value of the quantity lies in the interval *a*- to *a*+. Then *uj* is approximately 1.48 *a*, where *a* = (*a*+ - *a*-)/2 is the half-width of the interval.**Normal distribution: "2 out of 3"** Procedure:  Model the input quantity in question by a normal probability distribution and estimate lower and upper limits *a*- and *a*+ such that the best estimated value of the input quantity is (*a*+ + *a*-)/2 (i.e., the center of the limits) and there are 2 chances out of 3 (i.e., a 67 % probability) that the value of the quantity lies in the interval *a*- to *a*+. Then *uj* is approximately *a*, where *a* = (*a*+ - *a*-)/2 is the half-width of the interval.**Normal distribution: "99.73 %"** Procedure:  If the quantity in question is modeled by a normal probability distribution, there are no finite limits that will contain 100 % of its possible values. However, plus and minus 3 standard deviations about the mean of a normal distribution corresponds to 99.73 % limits. Thus, if the limits *a*- and *a*+ of a normally distributed quantity with mean (*a*+ + *a*-)/2 are considered to contain "almost all" of the possible values of the quantity, that is, approximately 99.73 % of them, then *uj* is approximately *a*/3, where *a* = (*a*+ - *a*-)/2 is the half-width of the interval.**Uniform (rectangular) distribution** Procedure:  Estimate lower and upper limits *a*- and *a*+ for the value of the input quantity in question such that the probability that the value lies in the interval *a*- and *a*+ is, for all practical purposes, 100 %. Provided that there is no contradictory information, treat the quantity as if it is equally probable for its value to lie anywhere within the interval *a*- to *a*+; that is, model it by a uniform (i.e., rectangular) probability distribution. The best estimate of the value of the quantity is then (*a*+ + *a*-)/2 with *uj* = *a* divided by the square root of 3, where *a* = (*a*+ - *a*-)/2 is the half-width of the interval.**Triangular distribution** The rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that values of the quantity in question near the center of the limits are more likely than values close to the limits, a normal distribution or, for simplicity, a triangular distribution, may be a better model. Procedure:  Estimate lower and upper limits *a*- and *a*+ for the value of the input quantity in question such that the probability that the value lies in the interval *a*- to *a*+ is, for all practical purposes, 100 %. Provided that there is no contradictory information, model the quantity by a triangular probability distribution. The best estimate of the value of the quantity is then (*a*+ + *a*-)/2 with *uj* = *a* divided by the square root of 6, where *a* = (*a*+ - *a*-)/2 is the half-width of the interval.  Schematic illustration of probability distributions The following figure schematically illustrates the three distributions described above: normal, rectangular, and triangular. In the figures, *µt* is the expectation or mean of the distribution, and the shaded areas represent ± one standard uncertainty *u* about the mean. For a normal distribution, ± *u* encompases about 68 % of the distribution; for a uniform distribution, ± *u* encompasses about 58 % of the distribution; and for a triangular distribution, ± *u* encompasses about 65 % of the distribution.  |