Lecture 5
Basic Counting Rule; Permutations; Combinations

Text: A Course in Probability by Weiss 3.1 ∼ 3.3

STAT 225 Introduction to Probability Models
January 26, 2014
Agenda

1. Review

2. Examples
Review

1. Basic Counting Rule
Review

1. Basic Counting Rule
2. Factorial
Review

1. Basic Counting Rule
2. Factorial
3. Permutation
Review

1. Basic Counting Rule
2. Factorial
3. Permutation
4. Combination
Review

1. Basic Counting Rule
2. Factorial
3. Permutation
4. Combination
5. Ordered Partition
Example 15

Illinois license plates consist of 4 digits followed by 2 letters. Whereas, in Ohio, license plates start with 3 letters and end with 4 digits. Assume all letters are capitals (without loss of generality).

1. For each state, how many possible license plates are there?
2. How many possible license plates are there for each state if no digit or letter is allowed to repeat?
3. How many possible license plates are there if they must have at least 1 vowel?
4. How many possible license plates are there if they must have at least one vowel or at least one 3?
Example 15

Solution.

1. IL: $26^2 \times 10^4$
   OH: $26^3 \times 10^4$

2. IL: $26 \binom{2}{10} 4 = 26 \times 25 \times 10 \times 9 \times 8 \times 7$
   OH: $26 \binom{3}{10} 4 = 26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7$

3. IL: $(26^2 - 20^2) \times 10^4$
   OH: $(26^3 - 20^3) \times 10^4$

4. IL:
   $(26^2 - 20^2) \times 10^4 + 26^2 \times (10^4 - 9^4) - (26^2 - 20^2) \times (10^4 - 9^4)$
   OH:
   $(26^3 - 20^3) \times 10^4 + 26^2 \times (10^4 - 9^4) - (26^3 - 20^3) \times (10^4 - 9^4)$
Example 16

Using a standard 52 card deck:

1. How many possible ways are there to get a 5 card poker hand?
2. What is the probability of getting a pair (with the other 3 cards different denominations)?
3. What is the probability of getting 2 pairs?
4. What is the probability of getting a full house?
5. What is the probability of getting a 3 of a kind (but not a full house)?
6. What is the probability of getting a straight?
7. What is the probability of getting a flush?
Example 16

Solution.

1. \[ \binom{52}{5} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} \]

2. \[ = \frac{13}{1} \left( \frac{4}{2} \right) \left( \frac{12}{3} \right) \left( \frac{4}{1} \right)^3 \frac{52}{5} \]

3. \[ = \frac{13}{2} \left( \frac{4}{2} \right)^2 \left( \frac{11}{1} \right) \left( \frac{4}{1} \right) \frac{52}{5} \]

4. \[ = \frac{13}{1} \left( \frac{4}{3} \right) \left( \frac{12}{1} \right) \left( \frac{4}{2} \right) \frac{52}{5} \]

5. \[ = \frac{13}{1} \left( \frac{4}{3} \right) \left( \frac{12}{2} \right) \left( \frac{4}{1} \right)^2 \frac{52}{5} \]

6. \[ = \frac{10}{1} \left( \frac{4}{1} \right)^5 \frac{52}{5} \]

7. \[ = \frac{13}{5} \left( \frac{4}{1} \right) \frac{52}{5} \]
Example 17

In a simplified version of the lottery, you have 20 numbers and 5 are drawn. You pick 5 numbers ahead of time and wait to see how many you matched those that were randomly drawn.

1. What is the probability you get 4 correct?
2. What is the probability you don’t get any correct?
3. What is the probability you get exactly 2 correct given you got at least 1 correct?
Example 17

Solution.

1 = \binom{5}{4} \binom{15}{1} \binom{20}{5}

2 = \binom{5}{0} \binom{15}{5} \binom{20}{5}

3 = \binom{5}{2} \binom{15}{3} \binom{20}{5} - \binom{15}{5} \binom{20}{5}
Example 18

Suppose Krannert only allows 5 spaces for a password to Portals. Suppose further you are only allowed to use a number or a letter, but the system is not case sensitive.

1. How many possible combinations are there?

2. If you cannot have 9 in the first space, how many possible combinations are there?

3. If you cannot have 9 in the first spot, what is the probability that all 5 blanks are odd numbers?

4. If you cannot repeat the same character, how many possible combinations are there?
Example 18

Solution.

1. \[(26 + 10)^5 = 36^5\]

2. \[(26 + 9) \times (26 + 10)^4 = 35 \times 36^4\]

3. \[= \frac{4}{35} \times \frac{5}{36}^4\]

4. \[=_{36} P_5 = 36 \times 35 \times 34 \times 33 \times 32\]