Lecture 9
Random Variables and Probability Mass Functions

Text: A Course in Probability by Weiss 5.1, 5.2, 5.8

STAT 225 Introduction to Probability Models
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Agenda

1. From Variables to Random Variables
2. Probability Mass Function
Variable
A variable denotes a characteristic that varies from one person or thing to another. Examples include height, weight, marital status, gender, etc. Variables can be either quantitative (numerical) or qualitative (categorical).

We use frequency and relative frequency when describing variables.

Example 25
The following is a chart describing the number of siblings each student in a particular class has. Note there are 40 students total.

<table>
<thead>
<tr>
<th>Siblings (X)</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>.200</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>.425</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>.275</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>.075</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>.025</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>1</td>
</tr>
</tbody>
</table>
Random Variables

A random variable is a real-valued function whose domain is the sample space of a random experiment. In other words, a random variable is a function

\[ X : \Omega \rightarrow \mathbb{R} \]

where \( \Omega \) is the sample space of the random experiment under consideration and \( \mathbb{R} \) represents the set of all real numbers.

Example 25 continued

We define the event \( A \) as the event that a student has 2 or more siblings. What is \( P(X \in A) \)?

Solution.
Types of Random Variables

There are two main types of quantitative random variables: **discrete** and **continuous**. A discrete random variable often involves a count of something. Examples may include number of cars per household, number of hours spent studying for a test, number of hours spent watching t.v. per day, etc.

**Discrete random variable**
A random variable $X$ is called a discrete random variable if the outcome of the random variable is limited to a countable set of real numbers (usually integers).

**Probability mass function (PMF)**
Let $X$ be a discrete random variable. Then the probability mass function (pmf) of $X$ is the real–valued function defined on $\mathbb{R}$ by

$$p_X(x) = P(X = x)$$

The capital letter, $X$, is used to denote random variable. Lowercase letter, $x$, is used to denote possible values of the random variable.
Example 26

Flip a fair coin 3 times. Let $X$ denote the number of heads tossed in the 3 flips. Create a pmf for $X$

Solution.

Basic Properties of a PMF
- $0 \leq p_X(x) \leq 1, \forall x \in \mathbb{R}$
- $\{x \in \mathbb{R} : p_X(x) \neq 0\}$ is countable
- $\sum_x p_X(x) = 1$
Example 27

Let $X$ be a random variable with pmf defined as follows:

$$p_X(x) = \begin{cases} k(5 - x) & \text{if } x = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

1. Find the value of $k$ that makes $p_X(x)$ a legitimate pmf.
2. What is the probability that $X$ is between 1 and 3 inclusive?
3. If $X$ is not 0, what is the probability that $X$ is less than 3?

Solution.
Summary
In this lecture, we learned
- Random variable
- Probability mass function (PMF) and its properties
- Probability computation with discrete random variables