Lecture 8

Bayes’ Rule

Text: A Course in Probability by Weiss 4.4

STAT 225 Introduction to Probability Models
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Agenda

1. Bayes’ Rule
2. Review

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Motivating example

Example (The Monty Hall Problem)

There was an old television show called Let’s Make a Deal, whose original host was named Monty Hall. The set–up is as follows. You are on a game show and you are given the choice of three doors. Behind one door is a car, behind the others are goats. You pick a door, and the host, who knows what is behind the doors, opens another door (not your pick) which has a goat behind it. Then he asks you if you want to change your original pick. The question we ask you is, “Is it to your advantage to switch your choice?”
The Monty Hall Problem

Solution.

Bayes' Rule

Let $A_1, A_2, \cdots, A_k$ form a partition of the sample space. Then for every event $B$ in the sample space,

$$P(A_j|B) = \frac{P(B|A_j) \times P(A_j)}{\sum_{i=1}^{k} P(B|A_i) \times P(A_i)} \quad j = 1, 2, \cdots, k$$
Example 24

Let us assume that a specific disease is only present in 5 out of every 1,000 people. Suppose that the test for the disease is accurate 99% of the time a person has the disease and 95% of the time that a person lacks the disease. What is the probability that the person has the disease given that they tested positive?

Solution.

Basic Concepts

- Random experiment: sample space, element, event
- Set operations: union, intersection
- Set relations: mutually exclusive, exhaust, partition
**Probability Rules**

- $0 \leq P(E) \leq 1$ for any event $E$, $P(\emptyset) = 0$, $P(\Omega) = 1$
- Complement rule: $P(A) = 1 - P(A^c)$
- General addition rule: 
  \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
- General multiplication rule: 
  \[ P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A) \]
- Conditional probability: 
  \[ P(A|B) = \frac{P(A \cap B)}{P(B)} \]
- Law of total probability: 
  \[ P(B) = \sum_{i=1}^{k} P(B \cap A_i) = \sum_{i=1}^{k} P(B|A_i) \times P(A_i) \]
- Independence: if $A$ and $B$ are independent, then 
  \[ P(A|B) = P(A), P(B|A) = P(B), \text{ and} \]
  \[ P(A \cap B) = P(A)P(B) \]

**Counting Problem**

- Basic counting rule
- Permutation
- Combination
- Ordered partition