Lecture 4
Basic Counting Rule; Permutations; Combinations
Text: A Course in Probability by Weiss 3.1 ∼ 3.3

STAT 225 Introduction to Probability Models
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Whitney Huang
Purdue University

Agenda
1 Basic Counting Rules
2 Permutations
3 Combinations
**Basic Counting Rule**

The Basic Counting Rule is used for scenarios that have multiple choices or actions to be determined.

**Factorial**

**Definition**

The factorial of a non-negative integer \( n \), denoted by \( n! \), is the product of all positive integers less than or equal to \( n \).

**Example:**

\[
5! = 5 \times 4 \times 3 \times 2 \times 1 = 120
\]

**Convention:**

\[
0! = 1
\]
Permutation

Definition (Permutation)

A permutation of \( r \) objects from a collection of \( n \) objects is any ordered arrangement of \( r \) distinct objects from the \( n \) objects.

- Notation: \((n)_r\) or \(nP_r\)
- Formula: \((n)_r = \frac{n!}{(n-r)!}\)

The special permutation rule states that anything permute it itself is equivalent to itself factorial.

Example:

\[(3)_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3! = 3 \times 2 \times 1 = 6\]
Combination

**Definition**

A combination of \( r \) objects from a collection of \( n \) objects is any unordered arrangement of \( r \) distinct objects from the \( n \) total objects.

**Remark:** The difference between a combination and a permutation is that order of the objects is not important for a combination.

- Notation: \( \binom{n}{r} \) or \( n \binom{r} \)
- Formula: \( \binom{n}{r} = \frac{n!}{(n-r)! \times r!} \)
Ordered partition

**Definition**

An ordered partition of $m$ objects into $k$ distinct groups of sizes $m_1, m_2, \ldots, m_k$ is any division of the $m$ objects into a combination of $m_1$ objects constituting the first group, $m_2$ objects comprising the second group, etc. The number of such partitions that can be made is denoted by $\binom{m}{m_1, m_2, \ldots, m_k}$.

**Remark:**
We called $\binom{m}{m_1, m_2, \ldots, m_k}$ the multinomial coefficient where

$$\binom{m}{m_1, m_2, \ldots, m_k} = \frac{m!}{m_1! \times \cdots \times m_k!}$$

**Example 13**

3 people get into an elevator and choose to get off at one of the 10 remaining floors. Find the following probabilities:

1. $P$ (they all get off on different floors)
2. $P$ (they all get off on the 5th floor)
3. $P$ (they all get off on the same floor)
4. $P$ (exactly one of them gets off on the 5th floor)
5. $P$ (at least one of them gets off on the 5th floor)

**Solution.**
Example 14

Suppose we have the fictional word “DALDERFARG”

1. How many ways are there to arrange all of the letters?
2. What is the probability that the 1st letter is the same as the 2nd letter?
3. What is the probability that an arrangement of all of the letters has the 2 D’s next to each other?
4. What is the probability that an arrangement of all of the letters has the 2 D’s next to each other and it has the 2 R’s grouped together (not necessarily the D’s and R’s next to each other)?
5. What is the probability that an arrangement of all the letters has the 2 D’s before the F?

Example 14

Solution.
Summary

In this lecture, we learned

- Basic counting rules
- Combinations and Permutations