Lecture 11
Bernoulli and Binomial Distributions

Text: A Course in Probability by Weiss 5.3

STAT 225 Introduction to Probability Models
February 16, 2014

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Agenda

1 Bernoulli Trials

2 Bernoulli/Binomial Random Variables
Bernoulli trials

Many problems in probability and its applications involve independently repeating a random experiment and observing at each repetition whether a specified event occurs. We label the occurrence of the specified event a success and the nonoccurrence of the specified event a failure.

Example:
Tossing a coin several times

Bernoulli trials cont’d

Bernoulli trials:
- Each repetition of the random experiment is called a trial
- We use $p$ to denote the probability of a success on a single trial

Properties of Bernoulli trials:
- Exactly two possible outcomes success and failure
- The outcomes of trials are independent of one another
- The success probability, $p$, and therefore the failure probability, $(1 - p)$, remains the same from trial to trial
**Bernoulli Random Variable**

Characteristics of the Bernoulli random variable:
Let $X$ be a Bernoulli r.v.

- The definition of $X$: It is the number of successes in a single trial of a random experiment
- The support (possible values for $X$): 0: “failure” or 1: “success”
- Its parameter(s) and definition(s): $p$: the probability of success on 1 trial
- The probability mass function (pmf):
  $$p_X(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$$
- The expected value: $E[X] = 0 \times (1-p) + 1 \times p = p$
- The variance:
  $$Var(X) = E[X^2] - (E[X])^2 = p - (p)^2 = p(1-p)$$

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**Binomial random variable**

We can define the Binomial r.v. as the number of successes in $n$ Bernoulli trials, where the probability of success in one trial is $p$

Characteristics of the Binomial random variable:
Let $X$ be a Binomial r.v.

- The definition of $X$: It is the number of successes in $n$ trials of a random experiment, where sampling is done with replacement (or trials are independent)
- The support: 0, 1, ⋯, $n$
- Its parameter(s) and definition(s): $p$: the probability of success on 1 trial and $n$ is the sample size
- The probability mass function (pmf):
  $$p_X(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, \cdots, n$$
- The expected value: $E[X] = np$
- The variance: $Var(X) = np(1-p)$
Example 27

To test for Extrasensory perception (ESP), we have 4 cards. They will be shuffled and one randomly selected each time, and you are to guess which card is selected. This is repeated 10 times. Suppose you do not have ESP. Let $R$ be the number of times you guess a card correctly. What are the distribution and parameter(s) of $R$? What is the expected value of $R$? Furthermore, suppose that you get certified as having ESP if you score at least an 8 on the test. What is the probability that you get certified as having ESP?

Solution.

Example 28

In Chris’ Stat 225 class, 75% of the students passed (got a C or better) on Exam 1. If we were to pick a student at random and asked them whether or not they passed. Let $X$ represent the number of student(s) who passed.

- What type of random variable is this? How do you know? Additionally, write down the pmf, the expected value, and the variance for $X$.
- Repeat under the following assumption: What about if we picked 10 students with replacement and let $X$ be the number of student(s) who passed.
Example 28 cont'd

Solution.

Example 29

Suppose that 95% of consumers can recognize Coke in a blind taste test. Assume consumers are independent of one another. The company randomly selects 4 consumers for a taste test. Let $X$ be the number of consumers who recognize Coke.

1. Write out the pmf table for $X$
2. What is the probability that $X$ is at least 1?
3. What is the probability that $X$ is at most 3?
Example 29 cont’d

Solution.

Summary

In this lecture, we learned

- Bernoulli trail: definition, properties
- Bernoulli/Binomial random variables: pmf, expected value, variance,