Lecture 27

Two-Way ANOVA: Interaction

STAT 512

Spring 2011

Background Reading

KNNL: Chapter 19

Topic Overview

- Review: Two-way ANOVA Models
- Basic Strategy for Analysis
- Studying Interactions

Two-way ANOVA

- Factor Effects Model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

where $\varepsilon_{ijk} \sim N\left(0, \sigma^2\right)$ are independent

and $\sum \alpha_i = \sum \beta_i = \sum (\alpha\beta)_{ij} = 0$

- SAS uses different constraints: $\alpha_a = 0$, $\beta_b = 0$, and $(\alpha\beta)_{ij} = 0$ for $i = a$ or $j = b$.

Constraints / Comparisons

- Under the usual constraints everything gets compared to the GRAND MEAN
- Under SAS constraints everything gets compared to the mean for the last level of each factor.
- In either case, differences are identified and contrasts can be used with the **results being exactly the same**. So in the big picture, you should be able to produce the basic estimates, but otherwise do not need to worry too much about the constraints.

Factor Effects

(under the textbook constraints)

- **Grand Mean**: Estimate μ by $\bar{Y}_{\bullet\bullet\bullet}$
- **Main Effects**
  - Estimate α_i by $\hat{\alpha}_i = \bar{Y}_{i\bullet\bullet} - \bar{Y}_{\bullet\bullet\bullet}$
  - Estimate β_i by $\hat{\beta}_j = \bar{Y}_{\bullet j\bullet} - \bar{Y}_{\bullet\bullet\bullet}$
- **Interaction**: Estimate $(\alpha\beta)_{ij}$ by

$$\widehat{(\alpha\beta)}_{ij} = \bar{Y}_{ij\bullet} - \bar{Y}_{i\bullet\bullet} - \bar{Y}_{\bullet j\bullet} + \bar{Y}_{\bullet\bullet\bullet}$$

General Strategy for Multiple ANOVA Analysis

- Every thing we are doing can be extended to any number of variables.
- We will now consider a general strategy for approaching this type of data.

General Strategy

1. Set up model with main effects and interaction(s), check assumptions, and examine interaction(s).
2. If no significant interaction, examine main effects individually, using appropriate adjustments for multiple comparisons, main effects plots, etc.
   - Note one could also possibly re-run the analysis without the interaction term (see section 19.1 in KNNL about pooling)

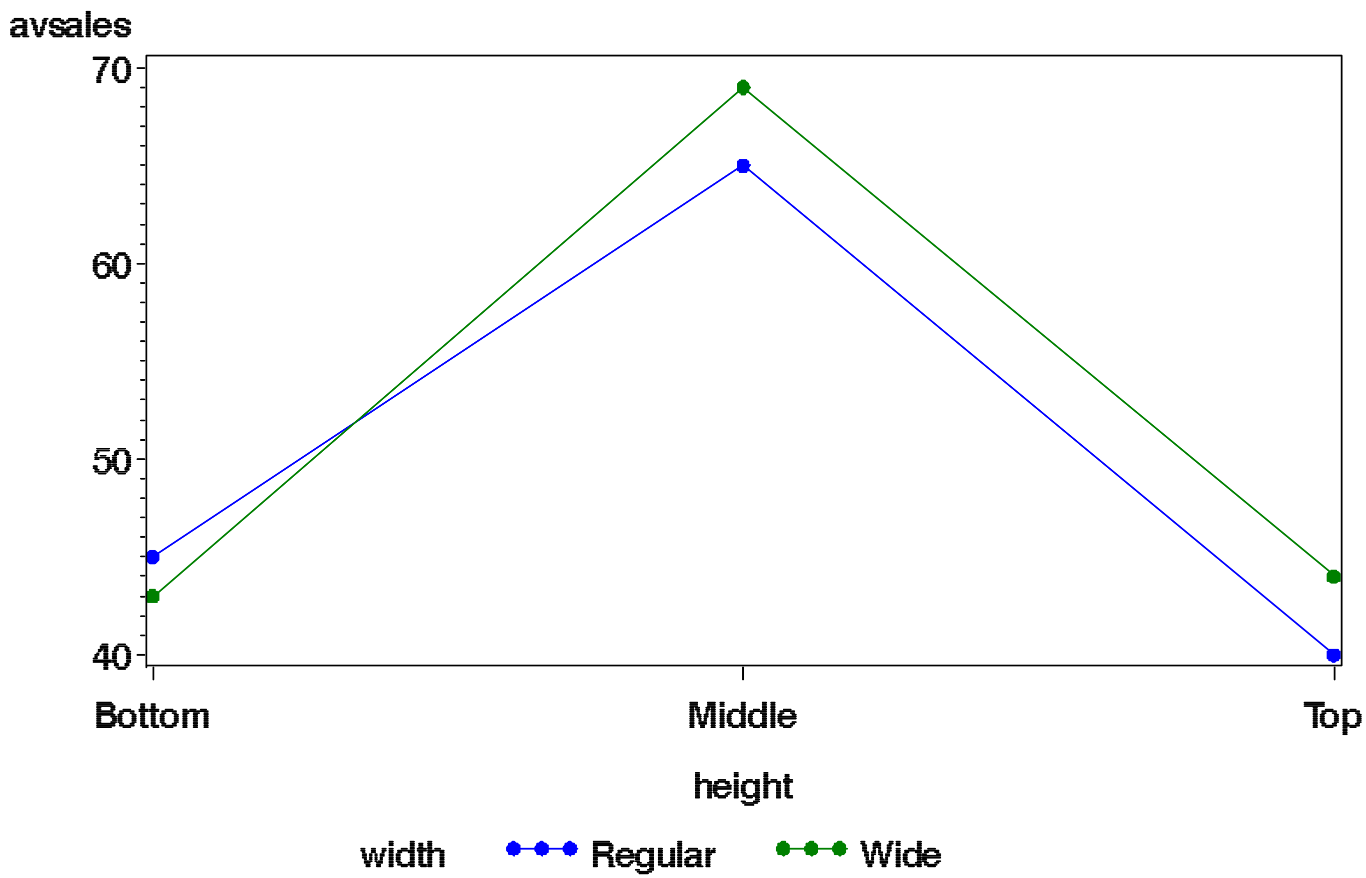
Analysis Strategies (2)

3. If interaction is significant, determine whether interactions are important. If not, can examine main effects as in Step 2.
4. If interaction present & important, determine whether interaction is simple or complex.
5. For simple interactions, can still talk about the main effects of A at each level of B
6. For complex interaction, must simply consider all pairs of levels as separate treatments.

Unimportant Interactions

- If interaction effects are very small compared to main effects or only apparent in a small number of treatments, then they are probably *unimportant*.
- Lines will be not quite parallel, but close.
- We can proceed by keeping interaction in the model, but using marginal means for each significant main effect individually
- Marginal means: Averages over the levels of the other factor.

Example (Unimportant Interaction)



Important Interactions

- The interaction effect is so large and/or pervasive that main effects cannot be interpreted on their own.

- In interaction plots, the lines will not be parallel. They may or may not criss-cross, but the differences between levels for one factor will *depend* on the level of the other factor

Important Interactions

Options include the following:

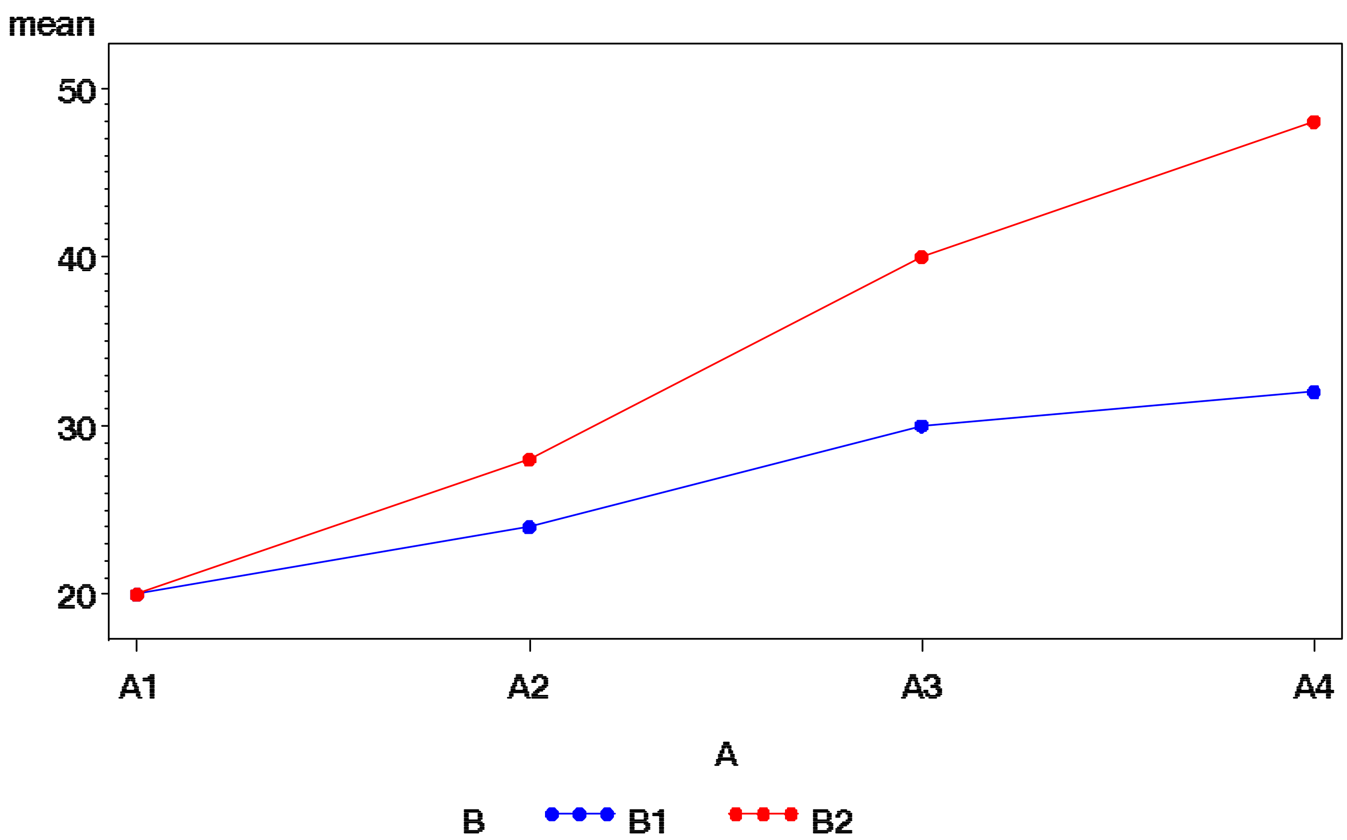
- Analyze interaction – Similar to interpreting as a one-way ANOVA with ab levels; use Tukey to compare means; contrasts and estimate can also be useful.
- Report that the interaction is significant; plot the means and describe the pattern.
- Discuss results for the levels of A for each level of B or vice versa

Simple vs. Complex Interactions

- An interaction is considered ***simple*** if we can discuss trends for the main effect of one factor for each level of the other factor, and if the general trend is the same.
- An interaction is ***complex*** if it is difficult to discuss anything about the main effects. In this situation, one can only look at treatment combinations and cannot separate them into main effects easily.

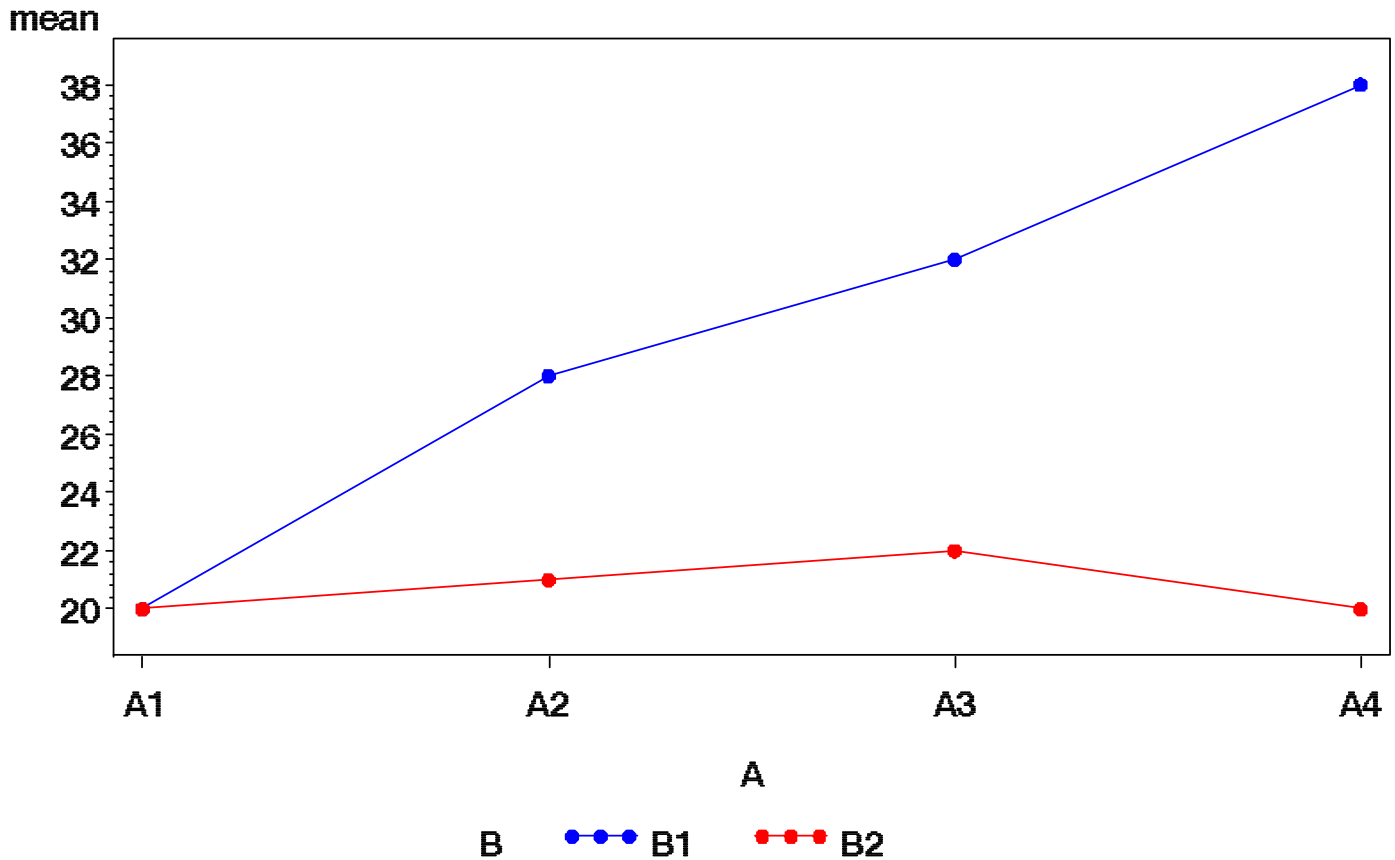
Example (Important Interaction)

Main effect of A is bigger for B = B2 than for B = B1



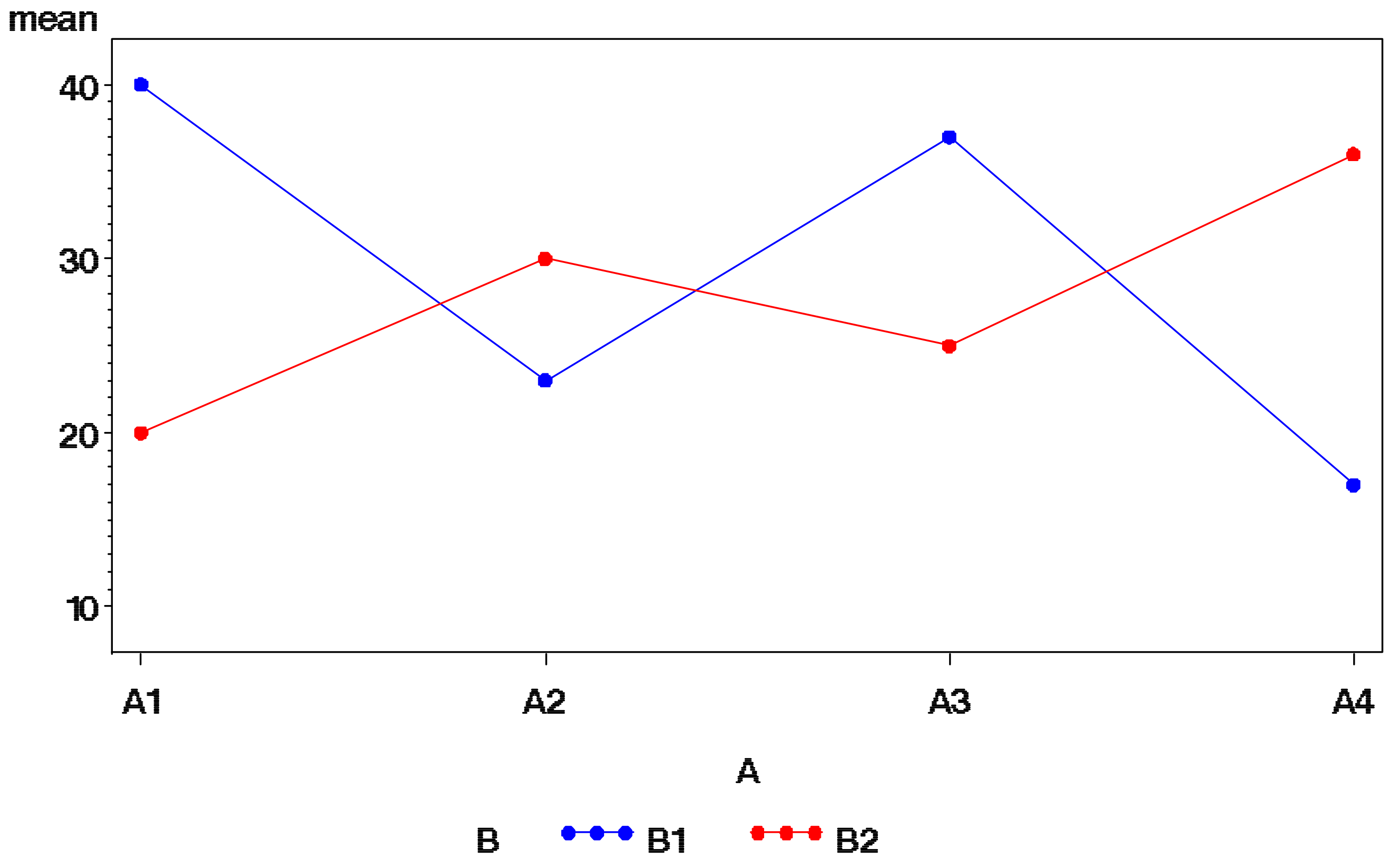
Example (Important Interaction #2)

Suppression: Effect of A is suppressed if B = B2



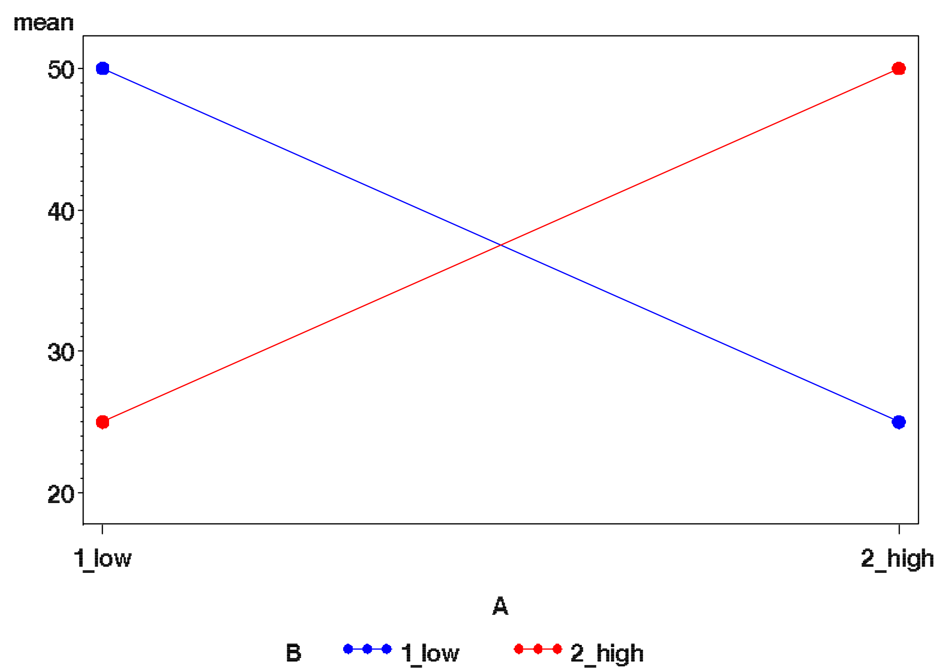
Example (Important Interaction #3)

Complicated Interaction: Cannot separate effects



Example (Important Interaction #4)

- In this case, main effects would “appear to be zero”. But this is misleading and inaccurate.



Example (Important Interaction #4)

- If you averaged over either factor, you would find “no change” when going from one level to the other.
- In fact there is a change when going from one level to the next, and the type of change depends on the level of the second factor. (This is a good “definition” for interaction.)

Example (Important Interaction #4)

Conclusions should be...

- At the low level of factor B, increasing A from low to high *decreases* the mean response.
- At the high level of factor B, increasing A from low to high *increases* the mean response.
- You cannot make statements here about Factor A alone or Factor B alone.

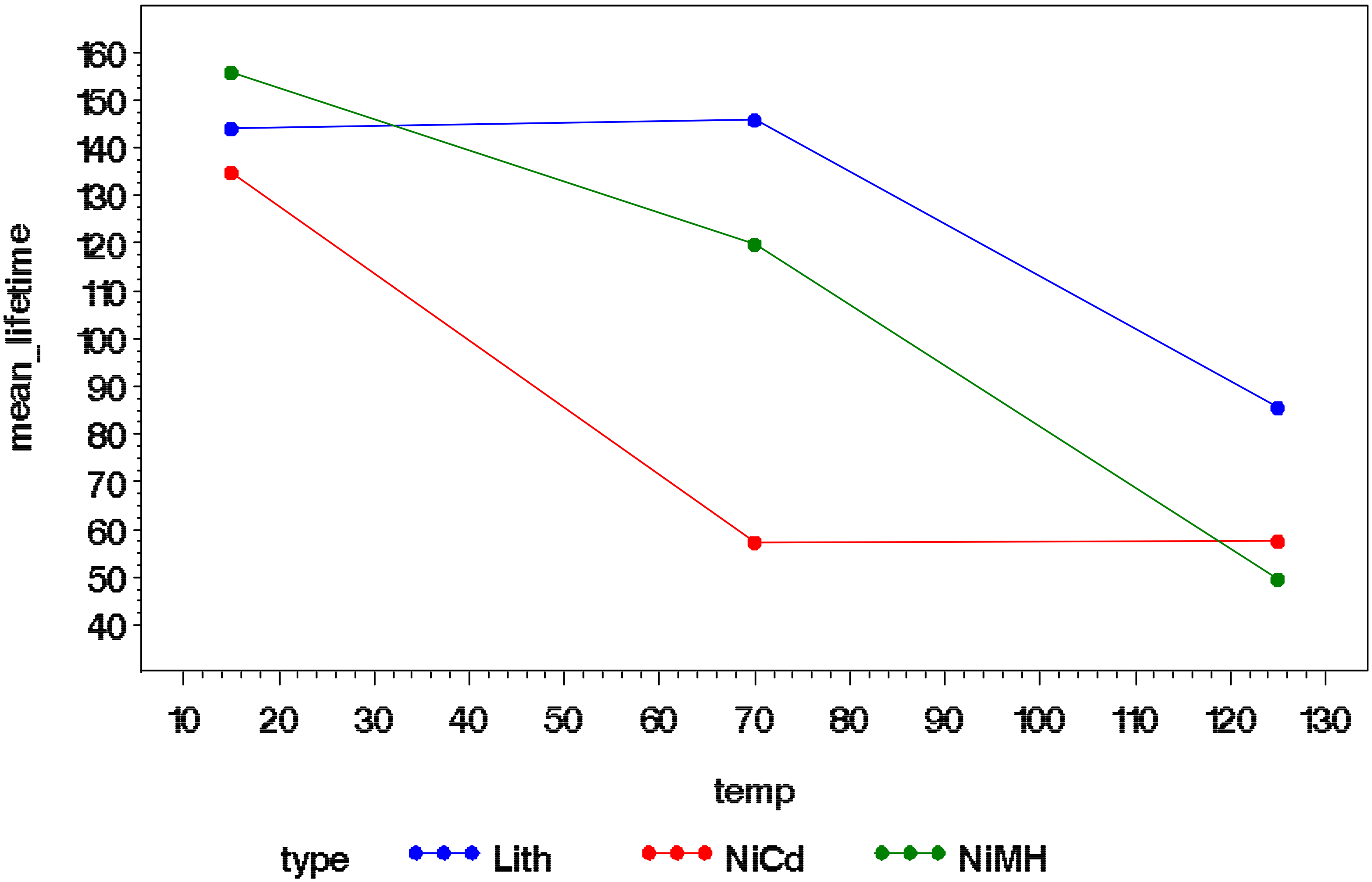
Battery Example

- Study the effects of A = type of material and B = temperature on the lifetime of a battery (in hours).
- Three material types (experimental) – Nickel-Cadmium, Nickel-Metal Hydride, and Lithium-Ion
- Three temperatures (also experimental) – 15, 70, and 125 degrees Fahrenheit

Battery Example (2)

- Four observations per cell
- Goal is to examine the effects and hopefully find a material that will help the battery have a uniformly long life in the field.
- Steps in analysis:
  - Check interaction plot
  - Review ANOVA results / assumptions
  - Check main effects if appropriate
  - Draw conclusions

Interaction

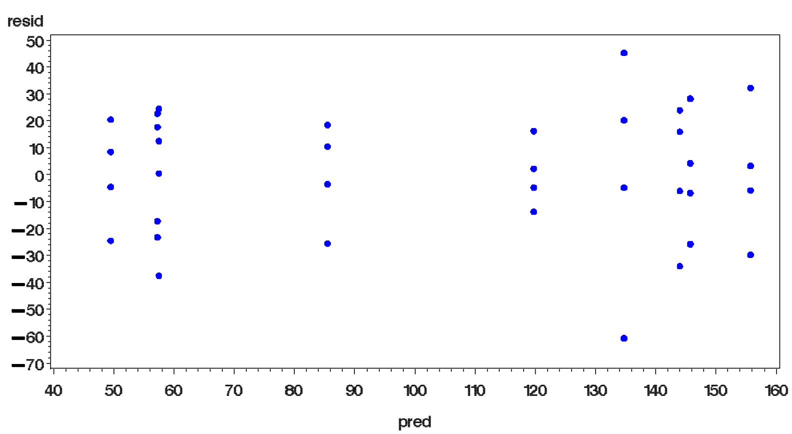


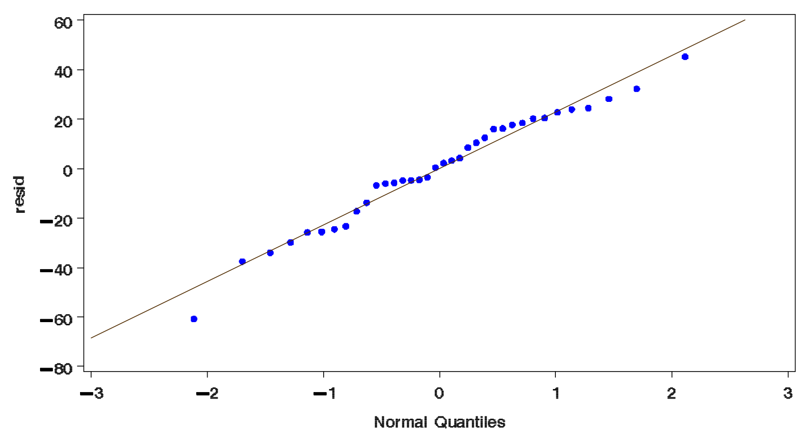
Interaction (2)

- Interaction here is complicated but quite informative.
- It appears the Ni-Cd battery is “worst” – we would want to eliminate that from production if the costs were all the same.
- We’ll take a look at the rest in greater detail, but the plot makes us suspect the Lithium ion battery is superior.

Assumptions

- No major violations of the assumptions are evident from reviewing the plots





ANOVA Results

Source	DF	SS	MS	F Value	Pr > F
type	2	10684	5342	7.91	0.0020
temp	2	39119	19559	28.97	<.0001
type*temp	4	9614	2403	3.56	0.0186
Error	27	18231	675		
Total	35	77648			

- All effects are significant, can look at multiple comparisons for type*temp.

LSMeans Output

type	temp	time LSMEAN	LSMEAN Number
Lith	15	144.000000	1
Lith	70	145.750000	2
Lith	125	85.500000	3
NiCd	15	134.750000	4
NiCd	70	57.250000	5
NiCd	125	57.500000	6
NiMH	15	155.750000	7
NiMH	70	119.750000	8
NiMH	125	49.500000	9

LSMeans Output (2)

i/j	1	2	3	4	5	6	7	8
2	1.0000							
3	0.0743	0.0604						
4	0.9999	0.9995	0.2017					
5	**0.0018**	**0.0014**	0.8282	**0.0065**				
6	**0.0019**	**0.0015**	0.8347	**0.0067**	1.0000			
7	0.9991	0.9997	**0.0172**	0.9616	**0.0003**	**0.0004**		
8	0.9165	0.8823	0.6420	0.9953	**0.0460**	**0.0475**	0.5819	
9	**0.0006**	**0.0005**	0.5819	**0.0022**	1.0000	1.0000	0.0001	**0.0172**

- Combine the p-values with the previous table to produce "groupings".

Tukey Groupings

type	temp	time LSMEAN	#	GRP
NiMH	15	155.75	7	A
Lith	70	145.75	2	A B
Lith	15	144.00	1	A B
NiCd	15	134.75	4	A B
NiMH	70	119.75	8	A B
Lith	125	85.50	3	B C
NiCd	70	57.25	5	C
NiCd	125	57.50	6	C
NiMH	125	49.50	9	C

Conclusions

- At 15 degrees there is no significant diff.
- At 70 degrees Lithium-ion or Nickel-MH are significantly different from Ni-Cd
- At 125 degrees there was no significant diff.
- If we had a little more power, we might be able to show that the Lithium-ion battery was best at 70 or 125 degrees, and equivalent to the others 15 degrees.
- Further testing with more observations might be useful.

Upcoming in Lecture 28...

- Additive Models
- One Case per Treatment
- Unequal Sample Sizes