Chapter 2
The Contribution to Experimental Designs by Kai-Tai Fang

Min-Qian Liu, Dennis K. J. Lin, and Yongdao Zhou

Abstract Professor Kai-Tai Fang has a wide research interest including applications of number-theoretic methods in statistics, distribution theory, experimental design, multivariate analysis and data mining. This paper only focuses on his contribution to experimental design. He proposed the method of visualization analysis for orthogonal designs in 1970. Inspired by three big military projects in 1978, he cooperated with Prof. Yuan Wang and proposed a new type of design of computer experiments, uniform design by utilized the number-theoretic methods. The uniform design can be also regarded as a kind of fractional factorial design, supersaturated design and design of experiments with mixture. In the past decades, the theory and applications of uniform designs have been developed rapidly by Kai-Tai Fang and his collaborators. In 2008, together with Professor Yuan Wang, Kai-Tai Fang received the 2008 State Natural Science Award at the Second Level, the highest level award in this kind of State award in that year. This paper focuses on the contribution of Kai-Tai Fang to experimental designs such as uniform designs, orthogonal designs, supersaturated designs and computer experiments.

2.1 Introduction

During the early 1970s, researches from Peking University and the Institute of Mathematics, Chinese Academy of Sciences, attempted to promote and apply orthogonal design to the industrial sector. In 1972, Kai-Tai Fang had the opportunity to go to the Tsingtao Beer Factory and other factories. He supervised their engineers to apply the

M.-Q. Liu · Y. Zhou (✉)
School of Statistics and Data Science, Nankai University, Tianjin 300071, China
e-mail: ydzhou@nankai.edu.cn

M.-Q. Liu
e-mail: mqliu@nankai.edu.cn

D. K. J. Lin
Department of Statistics, The Pennsylvania State University, University Park, PA 16802, USA
e-mail: dkl5@psu.edu

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orthogonal design to industrial experiments. During the consultancy process Kai-Tai Fang found that the engineers had difficulty to understand statistical methods, especially in calculating the ANOVA table without the help of computers or calculators in that time. Therefore, he realized the need for statisticians to simplify the complicated statistical theory and methods, and proposed the method of "Visualization Analysis" for analytical use on experimental data. Very soon this method was commonly used in the Mainland. He also suggested to use the range instead of sum of squares in ANOVA table, called as "the range analysis", see Fang and Liu [25]. The range analysis is simple to understand and easy to compute.

During the consultancy process Kai-Tai Fang met many case studies with multiple factors, large experimental domains and non-linear relationships between the response and factors. Some experiments can not reach the goal for several years. Faced with these complicated cases Kai-Tai Fang considered several issues: (1) the number of levels should be more than 2 (3–5 for example); (2) Considering all the possible factors in the first stage; (3) ranking importance of the factors and interactions for choosing recommended level-combination. By these considerations he helped the engineers to solve a number of complicated experiments. Kai-Tai Fang with his colleague Mr Liu summarized their experience into a Notes for giving lecture to engineers. Late, this Notes had been published in the journal, see Fang and Liu [24].

The most difficult problems Kai-Tai Fang met in 1978 can not be solved using the orthogonal designs. These problems gave a strong motivation for the establishment of the theory and method of uniform designs.

In summary, Kai-Tai Fang has authored and co-authored 25 monographs and textbooks, and published more than 300 papers, among which 5 monographs and more than 100 papers are on the research field of experimental designs. The purpose of this paper is to introduce Fang's contribution to uniform designs, orthogonal designs and supersaturated designs. The paper is organized as follows. Sections 2.2–2.5 introduce the contribution to uniform designs, orthogonal designs and supersaturated designs by Kai-Tai Fang, respectively. Some material is chosen from the paper "A Conversation with Kai-Tai Fang" by Loie et al. [50].

2.2 The Contribution to Uniform Designs

In 1978, Kai-Tai Fang took part in three major missile-related projects covering land, sea and aerospace. In these projects the true model between the response and factors can be numerical expressed by solving a system of differential equations. It needed a long computation time by a computer. It turned out the idea of computer experiments. Due to the Cultural Revolution there was no any information about the design of computer experiments from outside of China. Kai-Tai Fang and Yuan Wang considered to choose a certain number of experiments in the domain and find an approximate model to replace the true one. For example, one project needs a design with 6 factors some of which having at least 18 levels on a large experimental domain. Since the experiment was quite expensive and the speed of computer was quite slow (one experiment in one day), they wanted a design with at most 50 runs. Again, it was highly challenging. It needed a new method that could approximate a complicated system by a simple method with required accuracy. The great challenge was a motivating force to Kai-Tai Fang.

Kai-Tai Fang collaborated with Prof. Yuan Wang and borrowed the idea of number-theoretic methods to put experimental points uniformly on the domain and proposed the uniform design after a three-month hard working. Applying the uniform design to one of the three projects, 31 runs were arranged for the 6 factors each having 31 levels, and a satisfactory result was achieved. This method made that it was possible to calculate an accurate answer in 0.00001 s with the required accuracy. Eventually, the three projects were successful and won several nationwide awards. Kai-Tai Fang and Prof. Wang published two papers for introducing the uniform design theory in Chinese and English [4, 60], respectively. The new type of experimental designs was proposed since then. It was both time- and cost-saving and provided a valuable alternative design in computer experiments as well as laboratory experiments [17, 18, 23, 38]. During the 1970s, especially just after the Cultural Revolution in China, many scholars in China were still adhering to the modeling of the traditional experimental designs for data analysis, however, Kai-Tai Fang used regression analysis for modelling. Although the uniform design approach was not quite supported by few scholars in the experimental design, but it was greatly welcomed by the engineers. Several years later, the method of uniform designs has being used extensively in the mainland. Not only was it used for military purposes, but also it was adopted by and for civilians.

The idea of uniform design was from the overall mean regression model and the number-theoretic methods (Quasi-Monte Carlo methods). However, the uniformity is a geometric concept, not a statistical criterion. How to set up a solid theory is a very difficult target. Kai-Tai Fang had a difficult time during 1990–1996 after he moved to Hong Kong Baptist University. In fact, 90% of his academic pursuits has focused on uniform design since 1993. The progress was slow at the beginning. After several years, his collaboration with several scholars led to the discovery of a breakthrough.

In the following, we introduce the contribution to uniform designs by Kai-Tai Fang in the aspects of uniformity measures, construction methods of uniform designs and the relationship among different types of designs. Recently, Fang et al. [26] published a monograph that introduces the theory of the uniform design in details, and collects recent development in this direction.

2.2.1 Uniformity Measures

Assume $y = f(\mathbf{x})$ be the true model of a system on a domain $\mathscr{X} = C^s = [0, 1]^s = [0, 1] \times \cdots \times [0, 1]$, where $\mathbf{x} = (x_1, \ldots, x_s)$ are variables/factors and y is response. Let $\mathscr{P} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ be a set of n design points on C^s. One important issue is to estimate the overall mean of $f(\mathbf{x})$, i.e., $E(y) = \int_{C^s} f(\mathbf{x}) d\mathbf{x}$. A natural idea is to use the sample mean of $\mathscr{P}$, $\bar{y}(\mathscr{P}) = \frac{1}{n}\sum_{i=1}^{n} y_i$ to estimate $E(y)$, where $y_i = f(\mathbf{x}_i)$, $i = 1, \ldots, n$. The difference between $E(y)$ and the sample mean $\bar{y}(\mathscr{P})$ has following upper bound

$$|\bar{y}(\mathscr{P}) - E(y)| \leq V(f)D^*(\mathscr{P}), \tag{2.1}$$

where $V(f)$ is the total variation of the function f in the sense of Hardy and Krause (see Hua and Wang [45]; Niederreiter [54]), and $D^*(\mathscr{P})$ is the star discrepancy of $\mathscr{P}$ proposed by Weyl [63], which does not depend on f. The inequality (2.1) is the famous Koksma-Hlawka Inequality in quasi-Monte Carlo methods, and it is tight in some cases. If $V(f)$ is bounded in the experimental domain, then one may choose $\mathscr{P}$ with n design points on C^s such that its star discrepancy $D^*(\mathscr{P})$ is as small as possible and we can minimize the upper bound of the difference in (2.1). Fang [4] and Wang and Fang [60] called a design to be a uniform design if it has the smallest star discrepancy in the design space.

However, the star discrepancy has some shortcomings. Kai-Tai pointed out that it is not invariant under rotation of the coordinates, and is not easy to compute. He discussed this problem with his colleague Prof. Fred J. Hickernell. Hickernell [42, 43] used the tool of reproducing kernel Hilbert space, to generalize the definition of discrepancy and proposed different types of discrepancies. Among them the wrap-around L_2-discrepancy (WD) and centered L_2-discrepancy (CD) are popularly used. Fang et al. [17] gave the following requirements for a reasonable measure of uniformity.

C_1 It is invariant under permuting factors and/or runs.
C_2 It is invariant under rotation of the coordinates.
C_3 It can measure not only uniformity of $\mathscr{P}$ over C^s, but also the projection uniformity of $\mathscr{P}$ over C^u, where u is a non-empty subset of $\{1, \ldots, s\}$.
C_4 There is some reasonable geometric meaning.
C_5 It is easy to compute.
C_6 It satisfies the Koksma-Hlawka-like inequality.
C_7 It is consistent with other criteria in experimental design.

It has been known that the star discrepancy satisfies C_1, C_3, C_4 and C_6 and that both the WD and CD satisfy the requirements C_1–C_7. Later, Zhou et al. [70] considered the following two additional requirements for a uniformity measure.

C_8 Sensitivity on a shift for one or more dimensions.
C_9 Less curse of dimensionality.

Zhou et al. [70] also showed that CD does not satisfy the requirement C_9 and WD does not satisfy the requirement C_8. Then, they proposed another type of discrepancy, called mixture discrepancy (MD). The MD can satisfy C_1–C_9, which means that the MD can overcome the shortcomings of WD and CD, and MD may be the more reasonable measure of uniformity.

In many physical or practical situations, it prefers to have an experimental domain with a finite number of levels. Then, it is requested to give some discrepancies for experimental domain with finite candidates directly. Hickernell and Liu [44] and Fang et al. [20] proposed a discrepancy, called discrete discrepancy, which is also defined by a special kernel. Qin and Fang [57] further discussed the property of the discrete discrepancy and the construction methods of uniform designs. Besides, Zhou et al. [71] proposed the Lee discrepancy for finite numbers of levels. The discrete discrepancy is better for two-level designs and the Lee discrepancy can be used for multi-level designs.

It is known that a measure of uniformity plays a key role in the theory of uniform designs, Kai-Tai Fang, Fred J. Hickernell and their collaborators proposed different types of discrepancies, which greatly develop the theory of uniform designs. Based on those discrepancies, many relationships between uniform designs and other type of designs were shown by Kai-Tai Fang and his collaborators.

Given a type of discrepancies, a tight lower bound is useful for the construction of uniform designs, since it can be served as a benchmark during the searching procedure. Kai-Tai Fang and his collaborators gave many lower bounds for different types of discrepancies, see [28, 32, 35, 37].

2.2.2 Construction Methods of Uniform Designs

For the convenient use of uniform designs in practice, uniform design tables are very useful. Kai-Tai Fang and his collaborators Mingyao Ai, Gennian Ge, Fred J. Hickernell, Runze Li, Min-Qian Liu, Xuan Lu, Chang-Xing Ma, Jianhui Ning, Jianxin Pan, Hong Qin, Yu Tang, Yuan Wang, Xiaoqun Wang, Peter Winker, Aijun Zhang, Yongdao Zhou, etc., gave many construction methods, which include the following three approaches: (i) Quasi-Monte Carlo methods [4, 16, 73]; (ii) Combinatorial methods [9, 11–13, 13]; (iii) Numerical search [35, 64, 65, 68, 69].

The Quasi-Monte Carlo methods are popularly used to construct uniform designs, since the first group of uniform designs were generated from the number-theoretic methods. Among them, the good lattice point (glp) method and the glp method with power generator are firstly used by Fang [4]. The main idea of glp method for constructing an n-point s-factor design is to find a generator vector $(h_1, \ldots, h_s)$, where h_i is coprime with n and is $h_1, \ldots, h_s$ are different with each other. Then, the ith run of a glp set is determined by $d_{ij} = ih_j \pmod{n}$, which means a glp set is fully determined by the generator vector. One may find a best generator vector under some uniformity criterion. Moreover, given the parameters including the number of runs n and the number of factors s the uniformity of the design constructed by the glp method may have some space to improve. For example, based on a glp set, [73] showed that the linear level permutation technique can improve the space-filling property under the uniformity criterion and maximin distance criterion.

From 2000, Kai-Tai Fang began the collaboration with Gennian Ge from Suzhou University and Min-Qian Liu from Nankai University to link up combinatorial designs and uniform designs. Combinatorial construction methods are powerful to construct uniform designs under the discrete discrepancy, i.e., the resulting designs by those methods reach the minimum values of discrete discrepancy in many cases. The main tool of the combinatorial methods is the equivalence between an asymmetrical uniform designs with constant number of coincidences between any two rows and a uniformly resolvable design (URD). Therefore, given a URD, we can obtain a uniform design without any computational search. There are some miscellaneous known results on the existence of URDs, readers can refer to [11, 13] and the references therein for these results. The combinatorial methods can construct symmetric and asymmetric uniform designs, as well as supersaturated uniform designs. Some proposed construction methods by Kai-Tai Fang and his collaborators employed the following tools.

(A) Resolvable balanced incomplete block designs [9, 12, 13]
(B) Room squares [8]
(C) Resolvable packing designs [10, 27]
(D) Large sets of Kirkman triple systems [10]
(E) Super-simple resolvable t-designs [14]
(F) Resolvable group divisible designs [11]
(G) Latin squares [34]
(H) Resolvable partially pairwise balanced designs [36]

Here, (A)–(E) introduced the approaches for constructing symmetrical uniform designs, and (F)–(H) for asymmetrical cases. Most of those construction methods can obtain uniform designs under the discrete discrepancy.

The combinatorial methods only work for some special parameters n, s and $q_1, \ldots, q_s$. It is worth to give some construction methods of uniform designs for any given parameters. Kai-Tai Fang invited Peter Winker from Germany, a doctoral student then and a professor now, to cooperate for the numerical searching methods, which can satisfy such a requirement. Peter Winker is one of the experts on the threshold-accepting (TA) method. Winker and Fang [64] applied the TA for calculation of the star discrepancy and Winker and Fang [65] applied the TA for numerical searching uniform designs. This method uses the hard thresholds to accept the new solution in the neighborhood of current solution rather than some probability to accept the new solution in the simulation annealing method. Fang and Ma [29] and Fang et al. [31] used the TA algorithm to find uniform design tables under the WD and CD, respectively. Fang et al. [28] reexpressed the formulas of the WD and CD as functions of column balance, and also as functions of Hamming distances of the rows. And they also developed an efficient updating procedure for the local search heuristic threshold accepting based on these formulations of the WD and CD. Later, Fang et al. [35] proposed an efficient balance-pursuit heuristic algorithm to find many new uniform designs, especially with high levels. It was seen that the new algorithm is more powerful than the existing traditional threshold accepting algorithm. Fang et al. [32] also used the balance-pursuit heuristic algorithm to obtain many uniform designs. This algorithm uses some combinatorial properties of inner structures required for a uniform design. Moreover, Fang et al. [15] constructed uniform designs via an adjusted threshold accepting algorithm under the mixture discrepancy.

Later, Zhou et al. [69] reformed the optimization method for searching uniform designs into a zero-one quadratic integer program problem, and used some local searching methods to obtain the solution of such a problem, as well as the corresponding uniform design. Moreover, Fang et al. [23] found that many orthogonal designs can be generated by TA under the CD. Their results imply the so called "uniformly orthogonal design" by Fang and Ma [29], "Uniform fractional factorial designs" by Tang et al. [59].

2.3 More About Uniform Designs

In this section, more aspects of uniform designs are shown. We will show the contribution of Kai-Tai Fang on the topic of the connection between uniform designs and other types of designs, uniform designs for experiments with mixture and the application of uniform designs.

2.3.1 Connection Between Uniform Designs and Other Types of Designs

The uniform design theory was first proposed from Quasi-Monte Carlo method, and it is a deterministic method. It seems that the uniform design theory is totally different with orthogonal designs which have much statistical meaningfulness. Based on many research results of uniform designs, Kai-Tai Fang came up with the conjecture that most orthogonal designs are uniform in a certain sense. If this conjecture is true, we could link up orthogonal design with uniform design and obtain a vast development potential for uniform designs.

Kai-Tai Fang collaborated with several scholars and led to the discovery of a breakthrough. First, Kai-Tai Fang and Peter Winker found that such a conjecture was true in many cases, i.e., many existing orthogonal designs are also uniform designs. The result is based on the measure of uniformity proposed by Fang's colleague, Fred J. Hickernell. This discovery was of mutual benefit to both Hickernell and Fang. For Hickernell, his proposed measure of uniformity was initially not appreciated by many researchers in Quasi-Monte Carlo field but his measure became important in theory of uniform designs. For Fang, the measure of uniformity helped to prove that many existing orthogonal designs are uniform designs.

It still had one step to complete the proof of such a conjecture, i.e., we need a mathematical proof. Then, Kai-Tai Fang invited Rahul Mukerjee, Professor of the Indian Institute of Management in Calcutta, to HKBU for the collaboration in this topic. Rahul is a worldwide expert in the filed of experimental design. After two weeks, Rahul told Kai-Tai that the conjecture is not always true, even for a two-level factorial case. However, he showed an excellent result that it exists some relationship between uniformity and orthogonality. Usually, the wordlength pattern and the criterion "minimum aberration" are popularly used to measure the orthogonality of a regular design, and the CD can be used to assess the uniformity of a design. Kai-Tai and Rahul established an analytic relationship between the CD and wordlength pattern for regular designs. This discovery was immediately published in a top statistical journal, Biometrika, see Fang and Mukerjee [33]. It opened up an entirely new area that linked up uniform design and factorial design, an area in which Kai-Tai Fang collaborated with Chang-Xing Ma and others, and published more than 20 papers during 1999–2004. For example, Ma et al. [53] showed that the equivalence between the uniformity and orthogonality is only true in some special cases.

Tang et al. [59] gave the relationship between the CD and the generalized wordlength pattern of a three-level fractional factorial design, and also showed that minimum aberration designs have low discrepancies on average. Later, Zhou and Xu [72] obtained the close relationship between any discrepancy defined by the tool of reproduced kernel Hilbert space and the generalized wordlength pattern, which can measure the orthogonality of a nonregular design.

Moreover, Zhang et al. [67] used the majorization framework to generalize and unify classical criteria for comparisons of balanced lattice designs, which include fractional factorial designs, supersaturated designs and uniform designs. Fang and Ma [30] showed the relationship between uniformity, aberration and correlation in regular fractions 3^{s-1}. Furthermore, Ma et al. [52] used the CD to efficiently detect the isomorphism of fractional factorial designs.

Furthermore, the blocking design is an important type of experimental designs. Blocking experiments emphasize the balance among blocks, treatments or groups. Such a balance is easy to intuitively understand, and has a simple formula in data analysis. However, it needs to be proven in theory. Under the guide of Prof. Kai-Tai Fang, Liu and Chan [46] used the discrete discrepancy to prove that balanced incomplete blocking designs are the most uniform ones among all binary incomplete block designs. Liu and Fang [47] considered a certain kind of resolvable incomplete blocking designs, obtained a sufficient and necessary condition for such a blocking design is the most uniform in the sense of a discrete discrepancy measure, proposed a construction method for such designs via a kind of U-type designs, and set up an important bridge between resolvable incomplete blocking designs and U-type designs.

2.3.2 Uniform Designs for Experiments with Mixture

Usually, the experimental domain of uniform designs is a hypercube. Kai-Kai Fang and Yuan Wang firstly considered uniform designs for experiments with mixture [38, 61], i.e., the experimental domain becomes a simplex. Later, Fang and Yang [39] discussed uniform designs of experiments with restricted mixtures.

For constructing uniform design of experiments with mixtures, the uniformity criterion should be given first. There are two types of uniformity criteria, indirect and direct methods. One indirect method for measuring the uniformity of designs with mixtures is to measure the uniformity of the corresponding design on the hypercube C^{s-1} by a special transformation, see the F-discrepancy in Fang and Wang [38]. Ning et al. [56] proposed another uniformity criterion, DM2-discrepancy, for direct measuring the uniformity of designs with mixtures. Ning et al. [55] gave some construction method for the uniform designs with mixture on simplex.

2.3.3 Application of Uniform Designs

The achieved breakthrough in relation to uniform designs by Kai-Tai Fang and his collaborators won an international recognition. For example, the Handbook of Statistics (Volume 22) included the topic of uniform designs as a chapter, see Fang and Lin [19]. The Encyclopedia of Statistics Science (Second Edition) had chosen the aspect of uniform design as an entry, see Fang [5]. Both the Handbook of Engineering Statistics and the International Encyclopedia of Statistical Science by Springer also invited Kai-Tai Fang to write a chapter on uniform design for engineers, see Fang and Chan [7] and Fang [6], respectively. Moreover, Encyclopedia on Statistics in Quality and Reliability also invited Kai-Tai Fang to introduce the topic of uniform experimental designs, see Fang and Hickernell [3]. Uniform designs also won national acclaim. The Uniform Design Association of China (UDAC), as a branch of the Chinese Mathematical Society, was founded in 1994. The UDAC organized the national conferences, training courses, workshops and other activities to meet the calls to promote the applications of uniform designs.

In application-wise, there were numerous successful applications of uniform designs in China. With the keyword “uniform design”, you can find thousands of published case studies from the academic database China national knowledge infrastructure (CNKI), which collects most of the important academic journals in China. The application of uniform designs by Ford Motor Co. Ltd in USA is an exemplary application of this method. In Ford, Dr. Agus Sudjianto introduced to Kai-Tai Fang that the technique had become a critical enabler for them to execute “Design for Six Sigma” to support the new product development, in particular, for the automotive engine design. Moreover, it was told that computer experiments using uniform designs have become the standard practices at Ford Motor Co. Ltd to support the early stage of the production design before the availability of the hardware. As a result, Fang et al. [17] published a textbook/monograph entitled “Design and Modeling for Computer Experiments”, in which many case studies were from the real cases in Ford Motor Co. Ltd. In 2001 the 50th Gordon Research Conference: the Statistics in Chemistry & Chemical Engineering invited the topic “Uniform design for simulation experiments” as one of the nine topics, and each topic was given 3.5 h for introduction and discussion. Kai-Tai Fang, Professors Dennis K. J. Lin and Yizhen Liang (a chemist) formed a panel for this topic.

From the website of CNKI, there are 5660 papers used uniform designs to solve their problems between the period 2000–2018, see Fig. 2.1. There are also more than

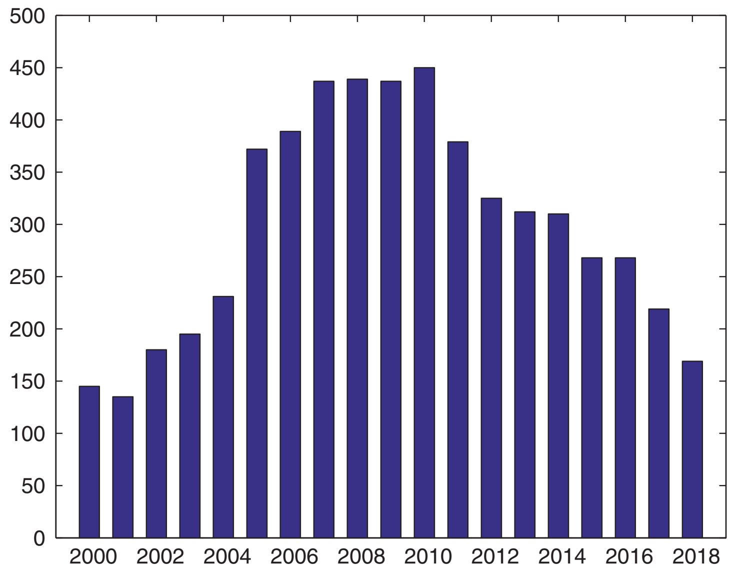

Fig. 2.1 The number of publications with the topic of uniform designs in CNKI

2000 citations of uniform designs from ISI Web of Science. Moreover, from the Google Scholar, the number of the citations of Kai-Tai Fang's publications is more than 14,000 times, and most of them are the citations of the papers about experimental designs, especially the topics of uniform designs.

2.4 The Contribution to Orthogonal Designs

During the process of promoting the common use of orthogonal designs, Kai-Tai Fang encountered quite a number of complicated multi-factor and non-linear issues. The engineers were unable to identify a satisfactory combination values of the parameters for a long time. An example was a porcelain insulator factory in Nanjing. The factory had a team whose job is to assign the conduction of the experiments continually for identifying a satisfactory combination values of the parameters. Although they achieved much knowledge in their experiments, they still failed to get a suitable combination of the values of the parameters to satisfy the requirement. At that time, the factory received a large number of orders for glass insulators but was unable to deliver the products. In view of the complexity of the issue, Kai-Tai Fang adhered to the principle of "big net catching big fish", and he conducted a 25-run experiment and arranged the six 5-level factors by an orthogonal design.

Such a design is a saturated design, which can not estimate all the main effects of the six factors, as well as none of the interaction effects can be estimated. However, in those 25 runs, all the responses of a special level-combination fulfill all the requirements. That was a great news to the factory in-charge. Should one liken the outcome to winning the US lottery or was it significant? In fact, using an orthogonal design to conduct 25 experiments actually represented 15,625 experiments, thus greatly increasing the probability of attaining an ideal technical/manufacturing condition.

The power of fractional factorial designs is that the experimental points have a good representation. Since then, Kai-Tai Fang used the same strategy to solve many of the "lasting, major and difficult" problems of the factories. This success encouraged Kai-Tai Fang to initiate the theory and method of uniform designs.

There are many criteria for assessing the property of orthogonal designs, such as minimum aberration [41], which is based on the wordlength pattern and can only be used for the comparison of regular designs. For extending such a criterion for nonregular designs, Kai-Tai Fang and Chang-Xing Ma used the MacWilliams identities to obtain the generalized wordlength pattern and the corresponding generalized minimum aberration criterion [51]. Independently, Xu and Wu [66] also obtained the generalized wordlength pattern by ANOVA models. The obtained generalized wordlength patterns by the two different ways are equivalent to each other for symmetrical nonregular designs. Additionally, the result in Xu and Wu [66] still works for asymmetrical designs. Later, Fang et al. [40] gave an effective algorithm for generation of factorial designs with generalized minimum aberration.

Moreover, Kai-Tai Fang cooperated with Lingyou Chan and Peter Winker to consider the relationship between orthogonal designs and optimal designs. They verified that each orthogonal array is an optimal design for a special polynomial regression models, see Chan et al. [1]. Liu et al. [49] showed the connections among different criteria for asymmetrical fractional factorial designs. Fang et al. [22] provided a theoretical justification for the optimal foldover plans for two-level designs, including the regular 2^{s-p}, nonregular, saturated and supersaturated designs.

2.5 The Contribution to Supersaturated Designs

A supersaturated design is essentially a fractional factorial design in which the number of potential effects is greater than the number of runs. A supersaturated design can be firstly used to screen the important factors in an experiment. Cooperated with Dennis K.J. Lin and Min-Qian Liu, Kai-Tai Fang gave a new criterion, $E(f_{NOD})$-criterion, for comparing supersaturated designs from the viewpoint of orthogonality and uniformity, see Fang et al. [20]. They also showed that the $E(f_{NOD})$-criterion is the generalization of the popularly used $E(s^2)$ and $ave\chi^2$ criteria for two- and three-level supersaturated designs, respectively. Moreover, Kai-Tai Fang also gave other criteria for assessing supersaturated designs such as $Ave(|f|)$, $Ave(f^2)$ and f_{max}, see Fang et al. [21].

Based on those criteria, Kai-Tai Fang and his collaborators gave many construction methods for multi-level and mixed-level supersaturated designs and investigated the properties of the obtained designs. The construction methods include the fractions of saturated orthogonal arrays (FSOA) method, the cyclic construction method, collapsing a U-type uniform design to an orthogonal array, and the global optimization algorithm, the threshold accepting algorithm, and the aforementioned combinatorial methods, see [2, 8, 10, 13, 14, 20, 21, 58]. Those results have high citations according to the ISI web of science. Moreover, Liu and Fang [48] used a uniform mixed-level supersaturated design to study a case in computer experiments, and explored the efficiency of supersaturated designs for screening important factors and building the predictors.

2.6 Conclusion

Kai-Tai Fang's contribution in the field of experimental designs includes the theoretical development and practical application of orthogonal designs, uniform designs and supersaturated designs. Moreover, he also has some contribution on other types of designs. For example, he showed that the optimal representative point method via quantizer is superior to using other methods (including orthogonal array) to design outer array points in Taguchi's product-array designs, see [62]. In a word, among his contributions, the most important one of Kai-Tai Fang is that he first proposed the uniform design with Yuan Wang. Uniform design becomes an important type of experimental designs which has great theoretical significance and application value. The uniform experimental design can be regarded as a fractional factorial design with model uncertainty, a space-filling design for computer experiments, a robust design against the model specification, a supersaturated design and can be applied to experiments with mixtures. Moreover, in the era of big data, experimental designs will also play an important role for the analysis of big data. Uniform designs also have such a chance to be used for dealing with their problem. For example, one can use uniform designs for the subsampling of big data.

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