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Theory of Factorial Design: Single- and Multi-Stratum Experiments. Ching-Shui Cheng. Boca Raton, FL: Chapman & Hall/CRC Press, 2013, xvi + 393 pp., $109.95 (H), ISBN: 978-1-46-650557-5.

The field of experimental design aims to help practitioners collect their data in a more efficient manner, or more specifically, run their experiments more effectively. There are many good textbooks in this area: the classical ones of the early 50's (e.g., Cochran and Cox 1957) focused more on agricultural experimentation; the later ones of the late 70's (e.g., Box, Hunter, and Hunter 1978) focused more on industrial experimentation, and the recent ones (e.g., Santner, Williams, and Notz 2003; Fang, Li, and Sudjianto 2006) focused more on computer experiments. There are also some theoretical approaches, notably on optimal design (e.g., Pukelsheim 1993) and combinatorics (e.g., Street and Street 1987). This (Cheng's) book is clearly one of the very first about design of experiment from a multi-stratum approach. I expect that it will prove to be an influential design book for years to come. The closest book is probably Bailey (2008), but Cheng's book is much more completed and updated.

In the preface, the author states that "*The objective of this book is to provide a rigorous, systematic, and up-to-date treatment of the theoretical aspects of this subject.*" and goes on to say "*A theory of orthogonal block structure that goes back to John Nelder provides a unifying framework for the design and analysis of multi-stratum experiments. One feature of the present book is to present this elegant and general theory which, once understood, is simple to use, and can be applied to various structures of experimental units in a unified and systematic way.*" This more or less indicates the uniqueness of this beautifully presented book. Some topics have never appeared in any other book and the author has produced elegant mathematics accompanied with lucid explanations.

The book is organized into 15 chapters. Chapter 1 provides an overview, which is very helpful for those who decide to carefully go through the entire book, or to use this book as a textbook. Chapters 2–5 are devoted to background material. It took me a while to get used to the notation used in the very brief Chapter 2 that contains a brief review of linear models that are used for the analysis and design criteria in later chapters. Chapter 3 (again very brief) introduces basic design concepts on randomization and blocking. Chapter 4 is rather unique in the way it makes use of proportional frequencies, first introduced in Chapter 2, to build a foundation for future chapters. The Hasse diagram is used here to illustrate block structure. I personally think Chapter 4's title "Factors" is somehow noninformative, however. Chapter 5 is the key component of the book. According to the author (p. 12), "*it [mainly Theorem 5.1] is to present a unified treatment of the analyses of three classes of orthogonal designs (completely randomized designs, complete block designs, and Latin square design) ... It is also a key result for developing a general theory of orthogonal designs for experiments with more complicated block structure.*" It is probably fair to say that Theorem 5.1 (p. 52) is the key theorem for the entire book. (Another key theorem is Theorem 13.2.) Theorem 5.1 is a powerful result and covers design construction for random effects, including fixed effect designs as special cases.

Chapters 6 and 7 are devoted to (full) factorial designs. Chapter 6 introduces treatment factorial designs through orthogonal polynomials, finite Euclidean geometry, and Abelian groups. Chapter 7 presents complete factorial designs in incomplete block, row-column layout, including split-plot and strip-plot designs. The concept of "design keys" is cleverly applied here.

Chapters 8–11 are devoted to fractional factorial designs. The combinatorial structure of orthogonal arrays is introduced in Chapter 8 and a brief connection to (recently popularized) computer experiments is discussed at the end of this chapter. Chapter 9 presents the regular fractional factorial designs, but uses a different set of notation than those used in previous chapters. Chapter 10 gives some recent advances in fractional factorial designs (mainly based on minimum aberration criterion). Chapter 11 focuses on Two-Level Resolution IV designs. While the content is good, many recent results have been missed, especially those pertaining to designs not of the foldover type; it seems to me that this topic is too narrow for a full chapter. The claim (p. 12) of "*a unifying framework for the design*" does not seem to fit these chapters particularly well.

Chapters 12–14 are devoted to factorial designs with more complicated block structures, namely, the multi-stratum designs. They connect nicely with Chapters 5–7 in terms of their content, approach, and notation. They are unique in that they are not included in other design books. Perhaps this is why they are mentioned in the title "Theory of Factorial Design: Single- and Multi-Stratum Experiments." Chapter 12 begins with a simple block structure. Chapter 13 then provides a general theory for multi-stratum complete factorial designs and details "design keys" in Section 13.6. Chapter 14 provides a rather comprehensive study on the construction of multi-stratum fractional factorial designs.

Chapter 15 is a relatively self-contained presentation of nonregular designs. Some recent interesting and important advances in this area are surveyed. As noted by the author that (p. xvi) "*The research on nonregular designs is still very active and expands rapidly. It deserves another volume.*"

I believe that this excellent book will soon become a must read for researchers and educators in experimental design. It could serve as a great reference or textbook for a high-level design course. Such a high-level design class, however, may be too specialized for most statistics education programs. For practitioners this book may be too theoretical: it would take some time to become familiar with its notation and concepts. I hope that this is a groundless concern from a person who has long known and respected the author. As claimed by the author, "*...once understood, (the notation) is simple to use, and can be applied to various structures of experimental units in a unified and systematic way.*" It would be ideal to implement the important concepts and theorems in this book into software so that they can be readily used for practitioners who do not typically have much theoretical background in design.

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