QUALITY ENGINEERING®
Vol. 16, No. 3, pp. 461–474, 2004

Application of Uniform Design in the Formation of Cement Mixtures

Ming Tang,[1] Jiukun Li,[2,3] Ling-Yau Chan,[3] and Dennis K. J. Lin[4,*]

[1]Department of Material Science and Engineering and [2]Department of Management Engineering, Shenyang Architectural and Civil Engineering Institute, University City, Huennan New District, Shenyang, People's Republic of China
[3]Department of Industrial and Manufacturing Systems Engineering, The University of Hong Kong, Hong Kong
[4]Department of Supply Chain and Information Systems, Pennsylvania State University, University Park, Pennsylvania, USA

ABSTRACT

As products and processes become more and more complex, there is an increasing need in the industry to perform experiments with a large number of factors and a large number of levels for each factor. For such experiments, application of traditional designs such as factorial designs or orthogonal arrays is impractical because of the large number of runs required. As an alternative, a type of design, called the uniform design, can be used to solve such problems. The uniform design has been intensively studied by theoreticians for several decades and has many successful examples of application in industry. In this article, we report a successful application of uniform design in product formation in the cement manufacturing industry. Specifically, we investigate the effects of additives on bleeding and compressive strength of a cement mixture. This example illustrates how an experiment of 16 runs was performed to study three factors with 16 levels, 8 levels, and 8 levels, respectively.

Key Words: Design of experiments; Uniform design; Factorial design; Orthogonal array; Experiments with mixtures.

*Correspondence: Dennis K. J. Lin, Department of Supply Chain and Information Systems, Pennsylvania State University, University Park, PA 16802-1989, USA; E-mail: DKL5@psu.edu.

DOI: 10.1081/QEN-120027947
0898-2112 (Print); 1532-4222 (Online)

1. DESIGN OF CEMENT MIXTURE

Cement matrix grouting material is commonly used in the construction industry, since it has high durability and high strength; also, the material is nontoxic, nonpolluting, and relatively low in cost. However, the disadvantages of ordinary cement matrix grouting material are its relatively low stability, low workability, and low water retentivity. This is especially true when the water/cement ratio is high, making it more prone to segregation and bleeding. These disadvantages hinder widespread use of this grouting material.

Experience shows that the presence of appropriate additives can improve the quality of this grouting material. Inorganic materials such as silica fume will increase the strength, water retentivity, and stickiness of the mixture, and reduce segregation; organic materials such as carboxyl methyl cellulose (CMC) will increase the stickiness of the mixture and thus reduce segregation; fly ash will increase the workability of the mixture. As fly ash is an industrial waste from thermal power stations, making use of it will help protect the environment. However, how much should each of these additives be added so that a cost-effective grouting material of good quality can be formed is a major concern to the manufacturer.

A project was conducted in a factory in Northeast China to find an optimal composition so that the cement grouting material formed has the desired properties. The objectives were to minimize the coefficient of bleeding BL (at water/cement ratio of 0.6) and maximize the compressive strength R_{28} (at water/cement ratio 0.8) which is measured twenty-eight days after the cement mixture has set. Four controllable variables were considered: percentages of fly ash, silica fume, CMC and cement, denoted by x_1, x_2, x_3 and x_4 respectively. It was decided to perform an experiment and to use the result obtained to construct regression models for BL and R_{28} in order to locate the optimal points. Since the factory had tight production schedules, very little resources were available for this project, and altogether only about 20 runs could be performed in the experiment.

Experience showed that ranges of x_1, x_2, x_3 in percentages should lie within the following ranges:

$$5 \le x_1 \le 20, \quad 1 \le x_2 \le 2.4, \quad 0.3 \le x_3 \le 1.0; \tag{1}$$

otherwise, either BL might be too large or R_{28} might be too low. Because of the constraint $x_1 + x_2 + x_3 + x_4 = 100$; the amount of cement x_4 should therefore lie within the range:

$$76.6 \le x_4 \le 93.7 \tag{2}$$

and any design layout can be expressed in terms of the variables x_1, x_2, x_3. An experiment with such a setup is referred to as an experiment with mixtures. For a review and a monograph of experiments with mixtures, see Chan (2000) and Cornell (2002). The design space of this experiment, which is a subregion of the following simplex,

$$S_{100}^{4-1} = \{(x_1, x_2, x_3)' : x_1, x_2, x_3 \ge 0, x_1 + x_2 + x_3 \le 100\}$$

is represented as the tetrahedron with vertices O, A, B, C in Fig. 1.

Experience shows that even slight changes of x_1, x_2, x_3, especially x_1, might have large effects on R_{28} and BL. In order to scrutinize the effects of these factors, designs of two or three levels were clearly insufficient. It was decided that the experiment should be performed with the following fine scales of division on x_1, x_2, x_3:

x_1: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20;

x_2: 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4;

x_3: 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.

Since x_1, x_2, x_3 are constrained by Eq. (1), only a small part of S_{100}^{4-1} is the admissible region for the experiment. In Fig. 1, the coordinates of O, A, B, and C are $(x_1, x_2, x_3)' = (0, 0, 0)'$, $(100, 0, 0)'$, $(0, 100, 0)'$, and $(0, 0, 100)'$, respectively. The five triangular planes $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$, $A_4B_4C_4$, and $A_5B_5C_5$, which are parallel to triangle ABC, correspond to $x_4 = 79.0$, 82.0, 85.0, 88.0 and 91.0, respectively. The admissible regions of x_1, x_2, and x_3 on these triangles are the parallelograms marked 1, 2, 3, 4, 5, respectively. The parallelogram marked i lies on the plane of triangle $A_iB_iC_i$ $(i = 1, 2, 3, 4, 5)$. The positions of the points A_i, B_i, C_i $(i = 1, \ldots, 5)$ in Fig. 1 are not to scale on the axes, nor are the positions and sizes of the five parallelograms.

2. UNIFORM DESIGN

Traditional designs such as fractional designs have been widely used in the industry (Dehnad, 1989; Montgomery, 2001). Designs of low levels (such as two or three levels) can be conveniently used to identify factors that have significant effects on the outcome. However, a restriction of such designs is their small number of levels on the factors, which

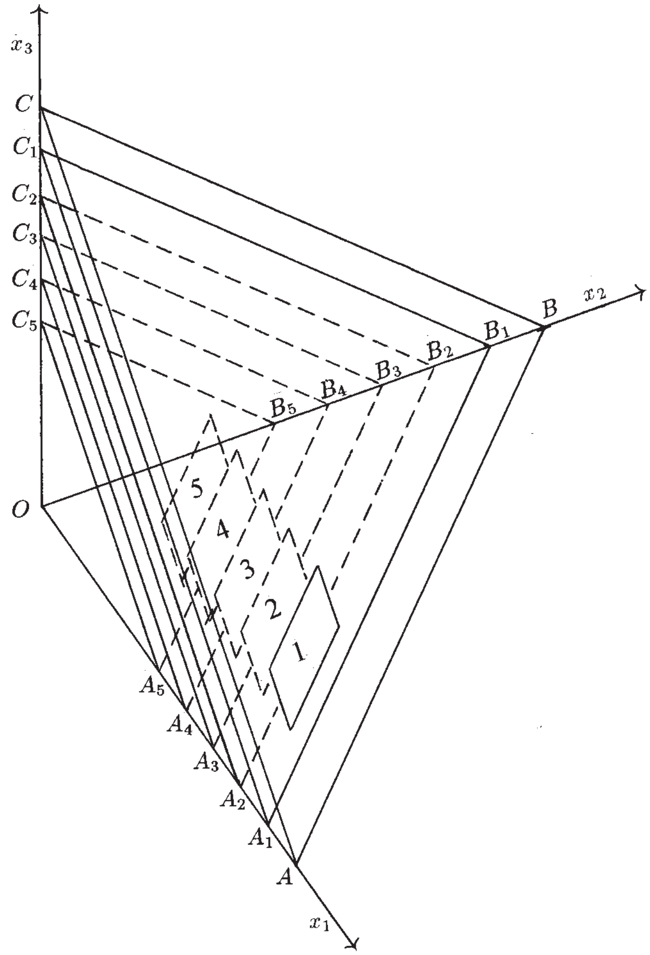

Figure 1. The tetrahedron with vertices A, B, C, O represents the region $S^{4-1} = \{(x_1, x_2, x_3, x_4) : x_1 + x_2 + x_3 + x_4 = 1;$ $x_i \geq 0\ (i = 1, 2, 3, 4)\}$.

does not provide reliable estimates of the slight changes on the response caused by fine variations in the design factors. If a larger number of levels is used, the experiment can be performed with better coverage of the design space but at the expense of a large number of runs. For example, the number of runs n of an $L_n(s_1^{r_1} \times s_2^{r_2})$ orthogonal array must be at least a multiple of $s_1^2 s_2^2$ when $r_1, r_2 \geq 2$. Even if s_1 and s_2 are of moderate sizes, say, $s_1 = 4$ and $s_2 = 5, s_1^2 s_2^2$ will be as large as 400, and such a number of runs usually cannot be realized in an actual industrial experiment.

The theoretical results in Fang and Mukerjee (2000) reveal that the success of factorial designs in exploration of the response surface is due to their uniformity in coverage of the whole design space, rather than their combinatorial or orthogonal property. If wide coverage of the design space is of primary importance, even when the number of levels of factors is large, designs with smaller numbers of runs can be constructed by sacrificing combinatorial properties of the design. A design without nice combinatorial properties can certainly serve the purpose of response surface analysis in industrial experiments, although such designs may not be desirable for ANOVA in some situations, for example, in surveys in social sciences research. The uniform design is such a design, which was first suggested in Wang and Fang (1981) and has been studied theoretically by many authors in the past several decades (Fang and Mukerjee, 2000; Hickernell, 1998, 1999; Liu and Hickernell, 2000; Wang and Fang, 1995). The uniform design can be easily adopted in practice, even if the number of levels of some factors is very large, say 20 or more. Readers are referred to Fang and Wang (1994) as a text reference; Fang et al. (2000) for a brief introduction to the basic theory, recent results, and an example; and Lee et al. (1997), Zhang et al. (1998), and Selected Papers (1997) for more examples of applications. Tables of uniform designs are available from the web site www.math.hkbu.edu.hk/Uniform Design/. This article reports a successful application of the uniform design in cement mixture formation, which cannot otherwise be carried out satisfactorily using traditional designs.

3. THE EXPERIMENT

The present experiment was performed with 16 levels in x_1 and 8 levels in each of x_2 and x_3, as shown in Sec. 1. An orthogonal array for this setup requires at least $16 \times 8^2 = 156$ runs, which is too large to be practical. A uniform design $U_{16}(16 \times 8^2)$ with 16 runs was used. The setup and the observed responses are shown in the first 6 columns and the first 16 rows of numbers in Table 1. The column BL contains the observed values of the coefficient of bleeding (in %) before the grouting process. The column R_{28} contains the observed compressive strength (in MPa) of the mixture 28 days after the cement has set. In Table 1, the numbers in parentheses (1), (2), etc., are the numbers representing the levels of factors in the $U_{16}(16 \times 8^2)$ uniform design table.

These data were fitted with the following Scheffé quadratic model (Scheffé, 1958):

$$\eta_2(\mathbf{x}) = \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \le i < j \le q} \beta_{ij} x_i x_j \tag{3}$$

where $\mathbf{x} = (x_1, \ldots, x_q)' \in S_{100}^{q-1}$, and $\mathbf{x}$ is under the constant sum constraint $x_1 + \cdots + x_q = 100$ $(q = 4)$. By the substitution $x_q = 100 - x_1 - \cdots - x_{q-1}$, the model $\eta_2(\mathbf{x})$ in Eq. (3) can be equivalently expressed in terms of $q-1$ regressor variables $x_1, \ldots, x_{q-1}$ as follows:

$$\begin{aligned}
\eta_2(\mathbf{x}) &= \sum_{i=1}^{q-1} \beta_i x_i + \beta_q(100 - x_1 - \cdots - x_{q-1}) \\
&\quad + \sum_{1 \le i < j \le q-1} \beta_{ij} x_i x_j \\
&\quad + \sum_{i=1}^{q-1} \beta_{i,q} x_i (100 - x_1 - \cdots - x_{q-1}) \\
&= \alpha_0 + \sum_{i=1}^{q-1} \alpha_i x_i + \sum_{1 \le i \le j \le q-1} \alpha_{ij} x_i x_j
\end{aligned} \tag{4}$$

Table 1. Results of the experiment.

Run number	x_1	x_2	x_3	BL	R_{28}	$[BL]$	$[R_{28}]$	e_{BL}	$e_{R_{28}}$
1	5 (1)	2.2 (7)	0.9 (7)	9.92	27.65	9.96	27.46	0.40	−0.69
2	6 (2)	1.8 (5)	0.8 (6)	11.26	26.58	11.35	26.24	0.80	−1.29
3	7 (3)	1.4 (3)	0.6 (4)	14.40	24.82	14.18	25.41	−1.55	2.39
4	8 (4)	1.0 (1)	0.5 (3)	16.03	23.15	16.02	23.62	−0.06	2.04
5	9 (5)	2.2 (7)	0.3 (1)	12.46	22.49	12.57	21.92	0.88	−2.52
6	10 (6)	1.8 (5)	1.0 (8)	4.85	26.06	4.87	26.01	0.41	−0.20
7	11 (7)	1.4 (3)	0.9 (7)	6.80	25.19	6.67	25.75	−1.91	2.24
8	12 (8)	1.0 (1)	0.7 (5)	9.95	28.02	10.21	26.45	2.61	−5.60
9	13 (9)	2.4 (8)	0.6 (4)	6.90	28.43	6.81	29.89	−1.30	5.14
10	14 (10)	2.0 (6)	0.4 (2)	10.34	22.08	10.13	21.95	−2.03	−0.57
11	15 (11)	1.6 (4)	0.3 (1)	12.40	17.15	12.51	17.07	0.089	−0.49
12	16 (12)	1.2 (2)	1.0 (8)	3.03	24.18	2.97	24.64	−1.98	1.91
13	17 (13)	2.4 (8)	0.8 (6)	3.51	30.56	3.51	29.78	0.00	−2.54
14	18 (14)	2.0 (6)	0.7 (5)	5.78	26.44	5.86	26.44	1.38	−0.01
15	19 (15)	1.6 (4)	0.5 (3)	9.73	20.35	9.88	20.01	1.54	−1.63
16	20 (16)	1.2 (2)	0.4 (2)	12.83	15.63	12.68	16.10	−1.17	3.04
1	5	1.0	0.4	19.62	20.6	20.31	19.83	3.52	−3.75
2	10	1.6	0.5	11.23	28.1	12.17	26.69	8.37	−1.47
3	15	1.8	0.8	4.91	28.7	5.20	29.59	5.90	−2.32
4	15	2.0	0.6	7.63	28.4	7.46	27.47	−2.23	−1.75
5	20	1.8	0.7	6.61	24.6	6.58	23.83	−0.45	−3.12

Table 2a. The ANOVA for BL in (3.5).

Source of variation	Sum of square	Degree of freedom	Mean square	F-ratio	Probability
Regression	1635.73	8	204.466	5168.85	1.69065×10^{-12}
Error	0.276902	7	0.0395574		
Total	1636.01	15			

Table 2b. The ANOVA for R_{28} in (3.6).

Regression	9686.41	8	1210.80	1187.09	2.9027×10^{-10}
Error	7.13981	7	1.01997		
Total	9693.55	15			

where the α's and β's are related by a one-to-one transformation. See Cornell (2002). A regression model selection procedure in the software package SAS was used to fit the data for BL and R_{28} in Table 1 using the form of $\eta_2(\mathbf{x})$ in Eq. (4). The results are expressed in the form of Eq. (3) as follows:

$$\begin{aligned} BL = {} & 3.3366x_1 - 341.32x_3 + 0.36548x_4 \\ & + 2.9980x_1x_3 - 0.045752x_1x_4 + 5.5116x_2x_3 \\ & - 0.056074x_2x_4 + 3.3229x_3x_4 \end{aligned} \tag{5}$$

$$\begin{aligned} R_{28} = {} & -6.6942x_1 + 3.4349x_2 - 4824.1x_3 \\ & - 0.58536x_1x_2 + 50.485x_1x_3 + 0.085773x_1x_4 \\ & + 61.285x_2x_3 + 48.526x_3x_4 \end{aligned} \tag{6}$$

Here the form in Eq. (3) was used, since it contains all the variables x_1, x_2, x_3, x_4 and is more convenient for the purpose of contour line plotting. In Eq. (5) and Eq. (6), all terms are significant at the 0.1 level. The values of R^2 and $C(p)$ of the regression equation in Eq. (5) are 0.999983 and 6.15525 and those in Eq. (6) are 0.999263 and 6.08415, respectively. The analysis of variance is shown in Table 2.

Equation (5) and Eq. (6) were verified by comparing the predicted values of BL and R_{28} with the observed values. The predicted values are in the columns under $[BL]$ and $[R_{28}]$ in Table 1, and the relative errors $e_{BL} = (BL - [BL])/BL$ and $e_{R_{28}} = (R_{28} - [R_{28}])/R_{28}$, in percentages, are shown in the last two columns in Table 1. Five additional runs were performed to validate the regression Eqs. (5) and (6), and the results obtained are shown in the last five rows in Table 1. Values of e_{BL} and $e_{R_{28}}$ in Table 1 show that the empirical models Eqs. (5) and (6) represent BL and R_{28} well.

4. BEHAVIOR OF *BL* AND R_{28}

In order to understand how BL and R_{28} change as $\mathbf{x}$ changes, contour lines are plotted on parts of the triangles $A_1B_1C_1, \ldots, A_5B_5C_5$ containing the admissible regions, which are the parallelograms marked $1, \ldots, 5$ in Fig. 1. Figures 2(a) through (e) show the contour plots of BL, and Figs. 3(a) through (e) show those of R_{28}.

Putting $x_4 = 100 - x_1 - x_2 - x_3$, BL in Eq. (5) will be represented as:

$$\begin{aligned} BL = {} & 36.548 - 1.604x_1 + 0.045752x_1^2 - 5.97288x_2 \\ & + 0.101826x_1x_2 + 0.056074x_2^2 - 9.3954x_3 \\ & - 0.279148x_1x_3 + 2.24477x_2x_3 - 3.3229x_3^2 \end{aligned} \tag{7}$$

where x_1, x_2, x_3 are free to vary in the region $\{(x_1, x_2, x_3)' \colon 5 \le x_1 \le 20,\ 1 \le x_2 \le 2.4,\ 0.3 \le x_3 \le 1.0\}$. Since it follows from Eq. (7) that $\partial BL/\partial x_2 = -5.97288 + 0.101826x_1 + 0.112148x_2 + 2.24477x_3 \le -5.97288 + 0.101826 \times 20 + 0.112148 \times 2.4 + 2.24477 \times 1 < 0$ and $\partial BL/\partial x_3 = -9.3954 - 0.279148x_1 + 2.24477x_2 - 6.6458x_3 < -9.3954 - 0.279148 \times 1 + 2.24477 \times 2.4 - 6.6458 \times 0.3 < 0$ in the region $\{(x_1, x_2, x_3)' \colon 5 \le x_1 \le 20, 1 \le x_2 \le 2.4, 0.3 \le x_3 \le 1.0\}$, the minimum value of BL in this region is attained when $x_2 = 2.4, x_3 = 1.0$. Setting $x_2 = 2.4$ and $x_3 = 1.0$ in Eq. (7) and minimizing BL shows that BL has a minimum value 0.5292 at $x_1 = 17.910$. Hence, up to two decimal places in the design region constrained by Eqs. (1) and (2), $\min(BL) = 0.52$ is attained at $(x_1, x_2, x_3, x_4) = (17.9, 2.4, 1.0, 78.7)$. The contour plots in Figs. 2(a) through (d) show how BL, represented by Eq. (7), changes as the variables as $x_1, \ldots, x_4$ change. These figures show that when

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

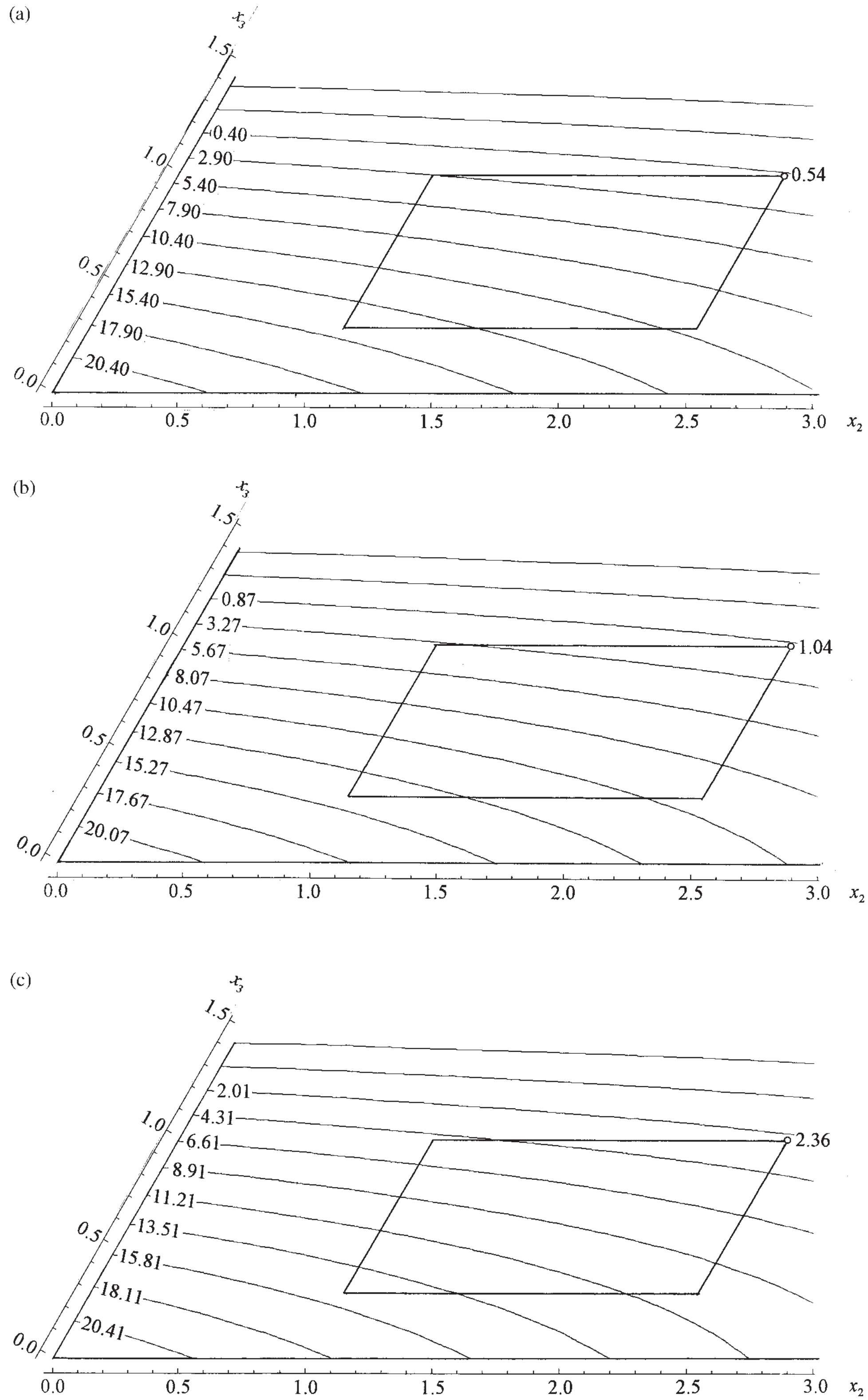

Figure 2. (a) Counter plots of BL on triangle $A_1B_1C_1$ ($x_4=79.0$); (b) counter plots of BL on triangle $A_2B_2C_2$ ($x_4=82.0$); (c) counter plots of BL on triangle $A_3B_3C_3$ ($x_4=85.0$); (d) counter plots of BL on triangle $A_4B_4C_4$ ($x_4=88.0$); (e) counter plots of BL on triangle $A_5B_5C_5$ ($x_4=91.0$).

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

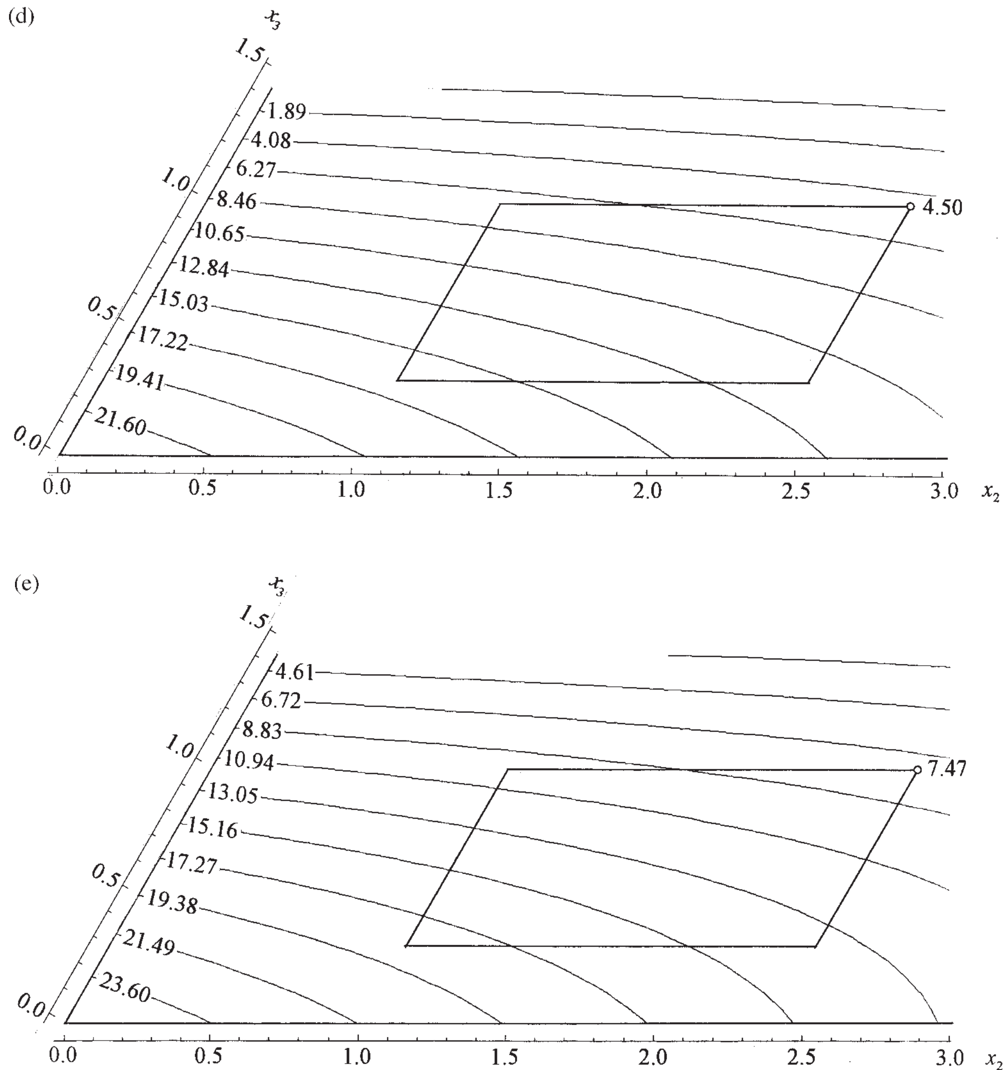

Figure 2. Continued.

$x_4 = 79, 82, 85, 88$, and 91, the minimum of BL is on the boundary of the design region where $x_2 = 2.4$ and $x_3 = 1.0$ (indicated by small circles in the figures), and this minimum increases as x_4 increases.

Putting $x_4 = 100 - x_1 - x_2 - x_3$, R_{28} in Eq. (6) will be represented as:

$$\begin{aligned} R_{28} = {} & 1.8831x_1 - 0.085773x_1^2 + 3.4349x_2 \\ & - 0.671133x_1x_2 + 24.491x_3 + 1.87323x_1x_3 \\ & + 12.759x_2x_3 - 48.526x_3^2 \end{aligned} \tag{8}$$

where x_1, x_2, x_3 are free to vary in the region $\{(x_1, x_2, x_3)': 5 \le x_1 \le 20, 1 \le x_2 \le 2.4, 0.3 \le x_3 \le 1.0\}$. From Eq. (8), putting $\partial R_{28}/\partial x_1 = \partial R_{28}/\partial x_2 = \partial R_{28}/\partial x_3 = 0$ gives $(x_1, x_2, x_3) = (16.6327, 0.244961, 0.605678)$, which is outside the admissible region $\{(x_1, x_2, x_3): 5 \le x_1 \le 20, 1 \le x_2 \le 2.4, 0.3 \le x_3 \le 1.0\}$. Hence, the maximum of R_{28} in this region must be attained on the boundary of the region. Finding the maximums of R_{28} on each of the faces, edges, and vertices of this region shows that the global maximum of R_{28} in this region is 33.6301, which is attained on the face $x_2 = 2.4$ at $(x_1, x_3) = (10.4302, 0.81068)$. Hence, up to two decimal places in the design region constrained by Eqs. (1) and (2), $\max(R_{28}) = 33.63$ is attained at $(x_1, x_2, x_3) = (10.43, 2.40, 0.81)$. The contour plots in Figs. 3(a) through (d) show how R_{28} represented by Eq. (8) changes as the variables

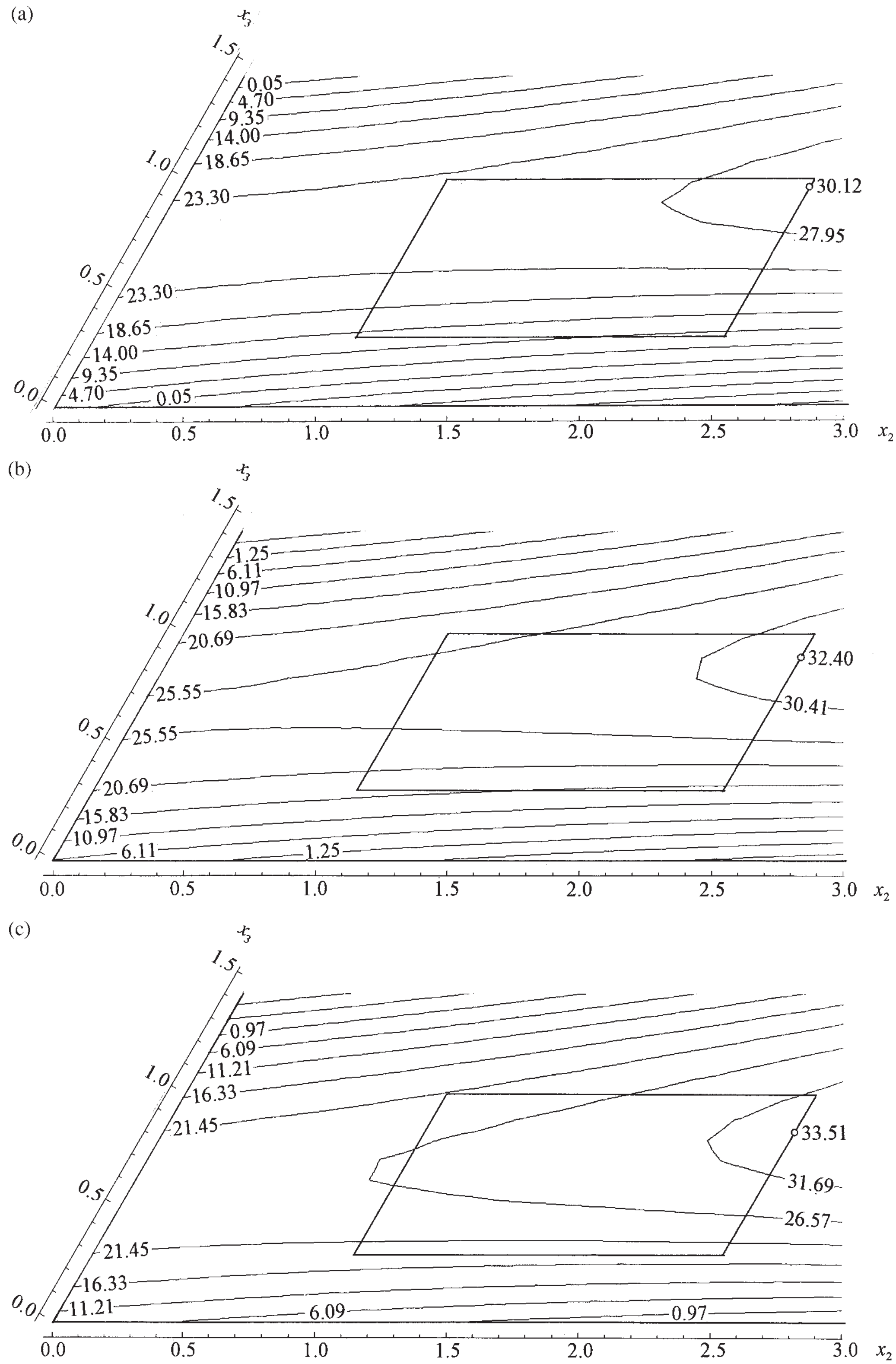

Figure 3. (a) Counter plots of R_{28} on triangle $A_1B_1C_1$ ($x_4 = 79.0$); (b) counter plots of R_{28} on triangle $A_2B_2C_2$ ($x_4 = 82.0$); (c) counter plots of R_{28} on triangle $A_3B_3C_3$ ($x_4 = 85.0$); (d) counter plots of R_{28} on triangle $A_4B_4C_4$ ($x_4 = 88.0$); (e) counter plots of R_{28} on triangle $A_5B_5C_5$ ($x_4 = 91.0$).

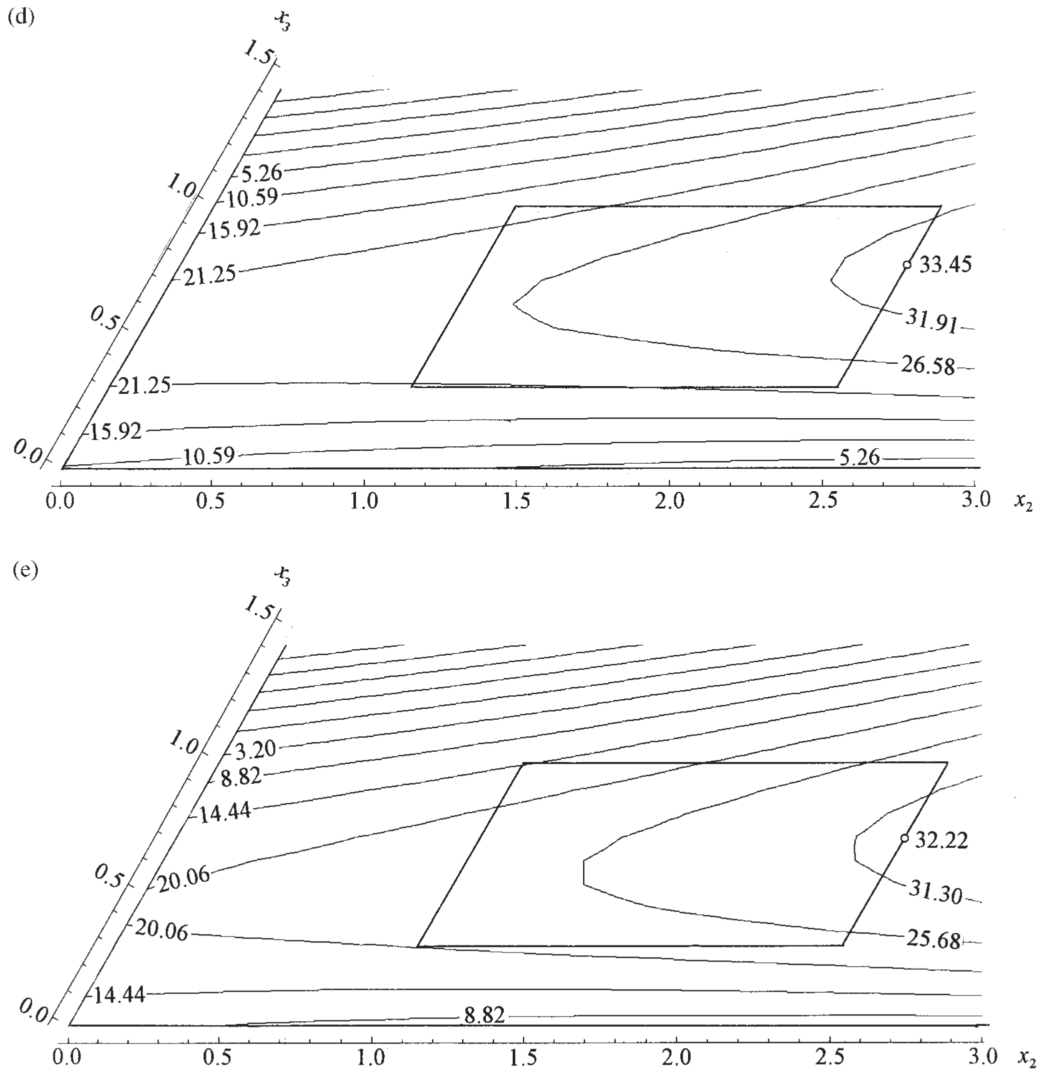

Figure 3. Continued.

$x_1, \ldots, x_4$ change. These figures show that when $x_4 = 79$, 82, 85, 88, and 91, the maximums of R_{28} occur when $x_2 = 2.4$ (indicated by small circles in the figures).

5. OPTIMIZATION

Analysis in the last section shows that in the admission region constrained by Eqs. (1) and (2), BL has a predicted minimum of 0.52 at $(x_1, x_2, x_3) = (17.9, 2.4, 1.0)$, while R_{28} has a predicted maximum of 33.63 at $(x_1, x_2, x_3) = (10.43, 2.4, 0.81)$, which is different from the minimum point of BL. As a trade-off, optimal compositions of the cement grouting mixture can be obtained by considering the material cost as well as the magnitudes of BL and R_{28}. The material costs of fly ash, silica fume, CMC, and cement are \$500, \$4000, \$5000, and \$320 per tonne (prices have been scaled here for confidentiality reason). The material cost per tonne for a mixture of composition $\mathbf{x} = (x_1, x_2, x_3, x_4)'$ is:

$$\begin{aligned} cost &= 5x_1 + 40x_2 + 50x_3 + 3.2x_4 \\ &= 5(100 - x_2 - x_3 - x_4) + 40x_2 + 50x_3 + 3.2x_4 \end{aligned} \tag{9}$$

The last equality follows from the constraint $x_1 + x_2 + x_3 + x_4 = 100$. Figures 3(a) through (e) show that within the range of experimentation, setting the amounts of silica fume and CMC at their maximums ($x_2 = 2.4, x_3 = 1.0$) gives the best values of *BL*, while setting x_2 at the maximum value 2.4 and setting $x_3 = 0.8$ (near the maximum value 1.0) yields the best value of R_{28}. However, these two additives are the most expensive among all ingredients. Optimal combinations of the amounts x_1, x_2, x_3, x_4 of the four ingredients—fly ash, silica fume, CMC, and cement—can be obtained by minimizing the *cost* in Eq. (4) under some conditions on *BL* and R_{28}, say $BL \leq 1$, 2, 3 and $R_{28} \geq 30, 25, 20$. This optimization can be performed with the aid of software such as MATLAB or Mathematica or done graphically as follows.

1. Substitute x_1 in Eq. (5) by $x_1 = 100 - x_2 - x_3 - x_4$, simplify the result, and express x_2 in terms of x_3, x_4, and *BL*:

$$x_2 = (-333.66 + BL + 44.8566x_3 + 2.998x_3^2 + 7.54632x_4 - 0.370752x_3x_4 - 0.045752x_4^2) \div (-3.3366 + 2.5136x_3 - 0.010322x_4) \quad (10)$$

2. Substitute x_2 in Eq. (10) into Eq. (9) to express *cost* in terms of x_3, x_4, and *BL*.
3. Substitute x_2 in Eq. (10) into Eq. (6) to express R_{28} in terms of x_3, x_4, and *BL*.
4. Use the result obtained in step 2 to plot contour maps of *cost* on the x_3x_4-plane for $BL = 1, 2, 3, 4$. These are shown in Figs. 4(a) through (d).
5. Use the result obtained in step 3 to obtain contour lines of $R_{28} = 20, 25, 30$ on the x_3x_4-plane and plot these contour line on

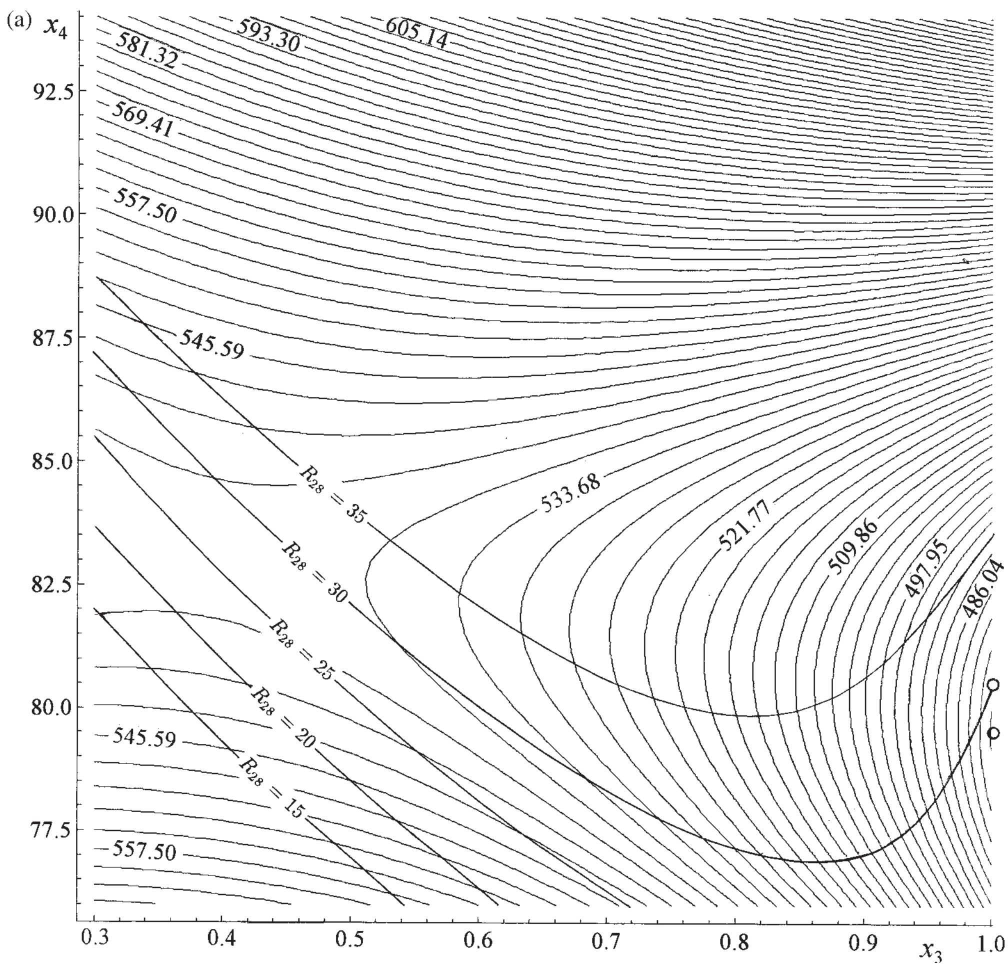

Figure 4. (a) Counter plots of *cost* for $BL = 1$; (b) counter plots of *cost* for $BL = 2$; (c) counter plots of *cost* for $BL = 3$; (d) counter plots of *cost* for $BL = 4$.

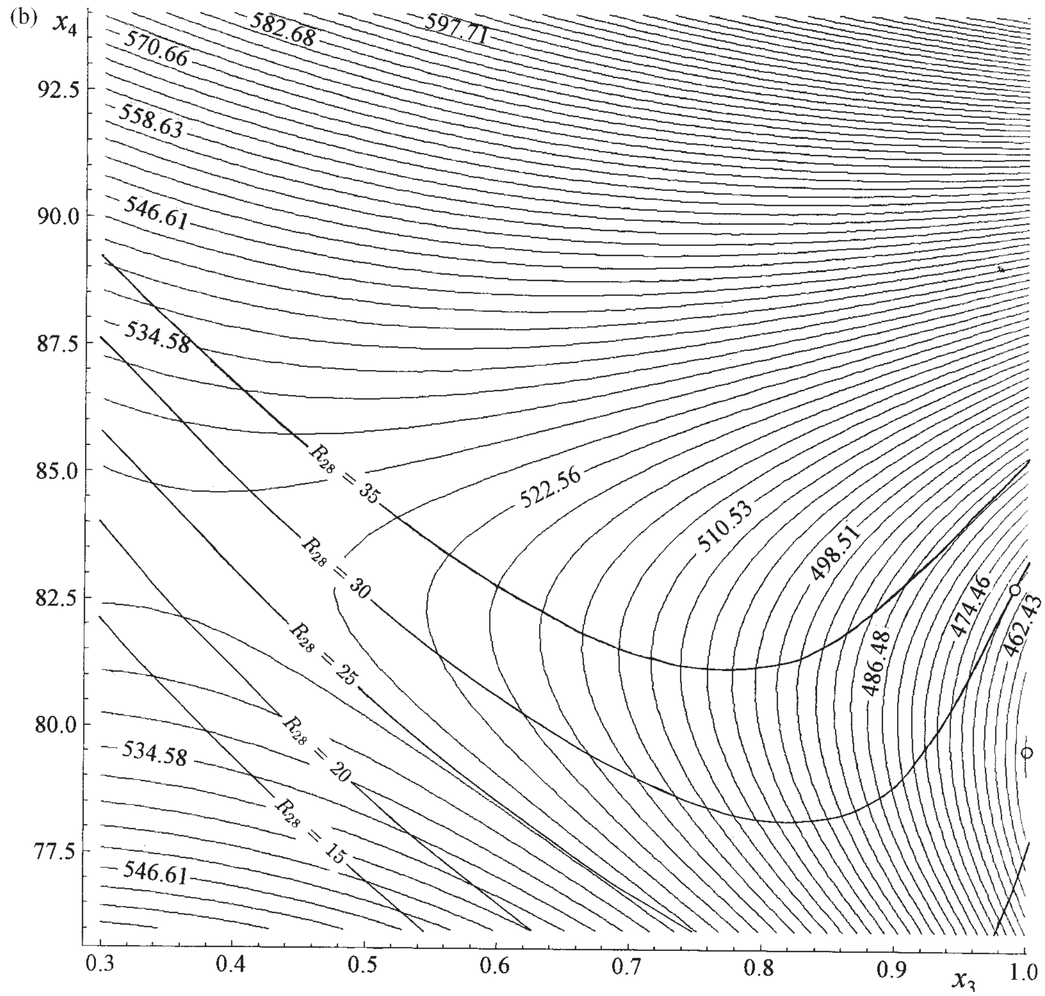

Figure 4. Continued.

(continued)

the contour maps of *cost* in Figs. 4(a) through 4(d). Locate the points on these contour maps with minimum values of *cost* that correspond to $R_{28} \geq 20, 25, 30$. These minimum points are indicated by small circles in Fig. 4(a) through 4(d). The results are shown in Table 3.

From Table 3, the following three grades of cements grouting material can be suggested:

Grade I: $(x_1, x_2, x_3, x_4) = (16.29, 2.21, 1.00, 80.50)$, $BL = 1.00$, $R_{28} = 30.007$, $cost = \$477.31$ (per tonne).

Grade II: $(x_1, x_2, x_3, x_4) = (17.87, 1.17, 0.99, 81.30)$, $BL = 3.00$, $R_{28} = 25.001$, $cost = \$440.46$ (per tonne).

Grade III: $(x_1, x_2, x_3, x_4) = (19.32, 0.30, 1.00, 79.38)$, $BL = 4.00$, $R_{28} = 21.499$, $cost = \$412.62$ (per tonne).

The contour plots in the figures suggest that some other values of the compositions (x_1, x_2, x_3, x_4) could give better values of BL and R_{28} than those of grade I mixture. As seen from Figs. 2(a) through 3(e), further decrease of BL and increase of R_{28} will force x_2 and x_3 to increase beyond their experimental limits of $x_2 \leq 2.4$ and $x_3 \leq 1.0$, respectively. However, extrapolating the empirical expression Eq. (7) for $x_2 > 2.4$ and $x_3 > 1.0$ will result in negative values of BL, which is physically impossible. This suggests that further experiments can be done to investigate how the cement mixture behaves when $x_2 > 2.4$ and $x_3 > 1$. By adopting segmented empirical models that take

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

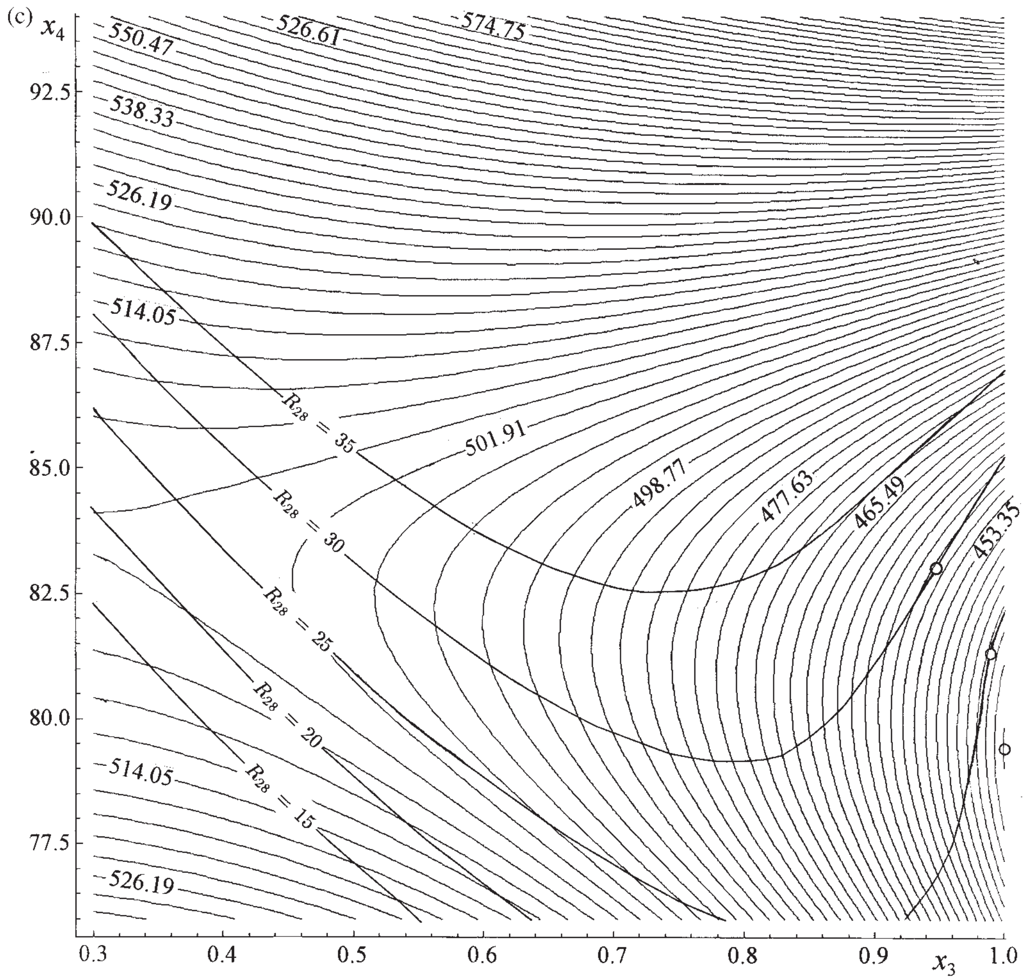

Figure 4. Continued.

the zero value (not negative value) over a region in the simplex and take positive values elsewhere in the simplex, a cement grouting material with $BL = 0$ and $R_{28} \gg 30$ may be obtained. Such a grouting mixture can be expected to contain higher proportions of silica fume and CMC and hence be more costly.

6. CONCLUSION

Factorial designs and orthogonal arrays have been widely used in design of industrial experiments. When the number of factors is large or the numbers of levels of the factors are large, these designs require a large number of runs, which may not be possible to achieve in practice because of various constraints. In such a case, the uniform design is an excellent alternative that can be used for the experiments. The uniform design has been studied extensively by mathematicians and statisticians for more than two decades, but its application in industries worldwide still has to be promoted. However, there have already been many successful applications of uniform designs in industry, especially in petroleum engineering, quality engineering, and system engineering. See Uniform Design Association of China (Selected Papers in Uniform Design, 1997) for a collection of published works.

This article gives a case study to illustrate how a uniform design is used in an experiment with 16 runs and 3 variables each having 16, 8, and 8 levels, respectively, in an experiment for cement mixture formation. In this case study, we provided a complete procedure for applying uniform design in product formation in industry—from design of experiment, to data analysis, to optimization. This can be easily extended and applied to other situations.

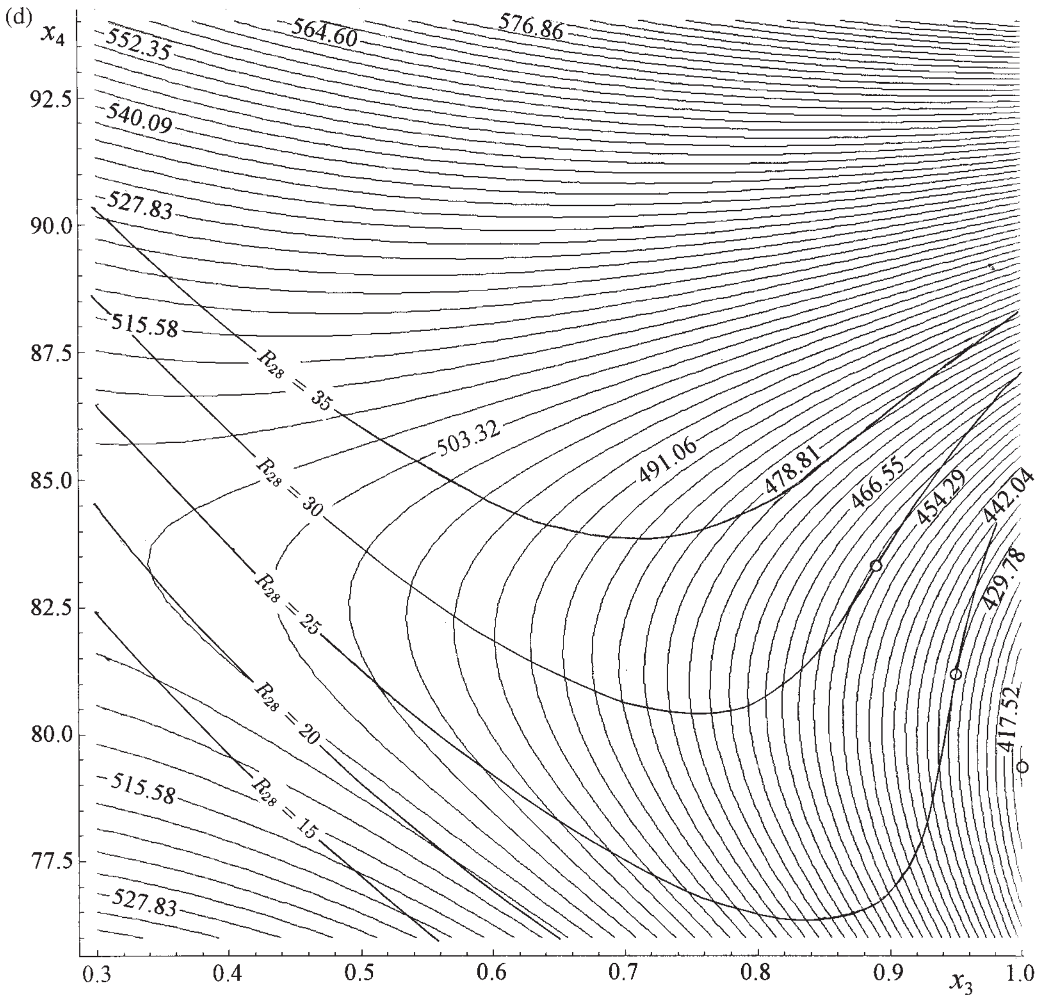

Figure 4. Continued.

Table 3. Minimum costs that correspond to $BL = 1, 2, 3, 4$ and $R_{28} \geq 30, 25, 20$.

BL	Region of optimization	Optimal point (x_1, x_2, x_3, x_4)	Value of R_{28}	Value of *cost* (minimum)
1.00	$R_{28} \geq 30$	$A = (16.29, 2.21, 1.00, 80.50)$	30.007	477.31
1.00	$R_{28} \geq 25$	$B = (17.28, 2.14, 1.00, 79.58)$	29.105	476.51
1.00	$R_{28} \geq 20$	$B = (17.28, 2.14, 1.00, 79.58)$	29.105	476.51
2.00	$R_{28} \geq 30$	$\mathrm{C} = (14.12, 2.08, 0.99, 82.81)$	30.003	468.12
2.00	$R_{28} \geq 25$	$D = (17.96, 1.52, 1.00, 79.52)$	26.083	455.22
2.00	$R_{28} \geq 20$	$D = (17.96, 1.52, 1.00, 79.52)$	26.083	455.22
3.00	$R_{28} \geq 30$	$E = (14.08, 1.97, 0.94, 83.01)$	30.002	461.88
3.00	$R_{28} \geq 25$	$F = (16.50, 1.21, 0.99, 81.30)$	25.001	440.46
3.00	$R_{28} \geq 20$	$G = (18.64, 0.91, 1.00, 79.45)$	23.551	433.92
4.00	$R_{28} \geq 30$	$H = (13.93, 1.89, 0.89, 83.29)$	30.008	456.18
4.00	$R_{28} \geq 25$	$I = (16.82, 1.03, 0.95, 81.20)$	25.002	432.52
4.00	$R_{28} \geq 20$	$J = (19.32, 0.30, 1.00, 79.38)$	21.499	412.62

ACKNOWLEDGMENT

This research is supported by grant number 59678053 from the National Science Foundation of China, Peoples' Republic of China.

ABOUT THE AUTHORS

Ming Tang is the dean and professor of the department of Material Science at Shenyang Architectural and Civil Engineering Institute, China. He has obtained more than 10 research grants, including one supported by the National Natural Science Foundation of China. He has published more than 60 papers in the fields of fractal, optimum experimental design and concrete science and has obtained the EUREK Aword Invention Fair Gold Medal in 2002.

Jiukun Li is an associate professor in the Department of Management Engineering, Shenyang Architectural and Civil Engineering Institute, China. She received a B.S. at Harbin Normal University in China and an M.Phil. at Hong Kong Baptist University. She has published more than 20 papers and is now the secretary in general of the Uniform Design Association of Liaoning Province.

Ling-Yau Chan is an associate professor and the head of the Department of Industrial and Manufacturing Systems Engineering at The University of Hong Kong. His research and consultancy areas include industrial statistics, design of experiments, statistical process control, reliability, quality management, and supply chain management. He has published more than 70 papers in these areas.

Dennis K. J. Lin is a professor of supply chain and statistics at the Pennsylvania State University, University Park. He received a Ph.D. in statistics from the University of Wisconsin at Madison. He is an elected Fellow of the American Statistical Association, an elected member of the International Statistical Institute, and a senior member of the American Society for Quality. He has published over 100 papers in the areas of design of experiments, response surface methodology, reliability, control charts, data mining, and statistical inference.

REFERENCES

Chan, L. Y. (2000). Optimal designs for experiments with mixtures: a survey. *Communications in Statistics—Theory and Methods* 29(9—10): 2281–2312.

Cornell, J. A. (2002). *Experiments With Mixtures*. 3rd ed. New York: John Wiley.

Dehnad, K. (1989). *Quality Control, Robust Design, and the Taguchi Method*. California: Wadsworth & Brooks/Cole.

Fang, K. T., Lin, D. K. J., Winter, P., Zhang, Y. (2000). Uniform design: theory and application. *Technometrics* 42:237–248.

Fang, K. T., Mukerjee, R. (2000). A connection between uniformity and aberration in regular fractions of two-level factorials. *Biometrika* 87:193–198.

Fang, K. T., Wang, Y. (1994). *Number-Theoretical Methods in Statistics*. London: Chapman & Hall.

Hickernell, F. J. (1999). Goodness-of-fit statistics, discrepancies and robust designs. *Statistics and Probability Letters* 44:73–78.

Hickernell, F. J. (1998). Lattice rules: how well do they measure up? In: Hellekalek, P., Larcher, G., eds. *Random and Quasi-Random Point Sets*, *Lecture Notes in Statistics*. Vol. 138. New York: Springer, 109–166.

Lee, A. W. M., Chan, W. F., Yuen, F. S. Y., Tse, P. K., Liang, Y. Z., Fang, K. T. (1997). An example of a sequential uniform design: application in capillary electrophoresis. *Chemometrics and Intelligent Laboratory Systems* 39:11–18.

Liu, M. Q., Hickernell, F. J. (2000). Connections between uniformity and $E(s^2)$—Optimality in 2-level supersaturated designs. In: Technical Report MATH-289, Hong Kong Baptist University, Kowloon Tong, Hong Kong.

Montgomery, D. C. (2001). *Design and Analysis of Experiments*. New York: John Wiley.

Scheffé, H. (1958). Experiments with mixtures. *Journal of the Royal Statistical Society, Series B* 20(2):344–360.

(1997). *Selected Papers in Uniform Design*. Beijing, People's Republic of China: Uniform Design Association of China, 2.

Wang, Y., Fang, K. T. (1981). A note on uniform distribution and experimental designs (in Chinese). *Kexue Tongba* 26:485–489.

Wang, Y., Fang, K. T. (1995). Uniform design of experiments with mixtures. In: Proceedings, Workshop on Quasi-Monte Carlo Methods and Their Applications. Baptist University, Hong Kong, December, 154–165.

Zhang, L., Liang, Y. Z., Jiang, J. H., Yu, R. Q., Fang, K. T. (1998). Uniform design applied to nonlinear multivariate calibration by ANN. *Analytica Chimica Acta* 370:65–77.