Application of Uniform Design in the Formation of Cement Mixtures

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ABSTRACT

As products and processes become more and more complex, there is an increasing need in the industry to perform experiments with a large number of factors and a large number of levels for each factor. For such experiments, application of traditional designs such as factorial designs or orthogonal arrays is impractical because of the large number of runs required. As an alternative, a type of design, called the uniform design, can be used to solve such problems. The uniform design has been intensively studied by theoreticians for several decades and has many successful examples of application in industry. In this article, we report a successful application of uniform design in product formation in the cement manufacturing industry. Specifically, we investigate the effects of additives on bleeding and compressive strength of a cement mixture. This example illustrates how an experiment of 16 runs was performed to study three factors with 16 levels, 8 levels, and 8 levels, respectively.

Key Words: Design of experiments; Uniform design; Factorial design; Orthogonal array; Experiments with mixtures.

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1. DESIGN OF CEMENT MIXTURE

Cement matrix grouting material is commonly used in the construction industry, since it has high durability and high strength; also, the material is nontoxic, nonpolluting, and relatively low in cost. However, the disadvantages of ordinary cement matrix grouting material are its relatively low stability, low workability, and low water retentivity. This is especially true when the water/cement ratio is high, making it more prone to segregation and bleeding. These disadvantages hinder widespread use of this grouting material.

Experience shows that the presence of appropriate additives can improve the quality of this grouting material. Inorganic materials such as silica fume will increase the strength, water retentivity, and stickiness of the mixture, and reduce segregation; organic materials such as carboxyl methyl cellulose (CMC) will increase the stickiness of the mixture and thus reduce segregation; fly ash will increase the workability of the mixture. As fly ash is an industrial waste from thermal power stations, making use of it will help protect the environment. However, how much should each of these additives be added so that a cost-effective grouting material of good quality can be formed is a major concern to the manufacturer.

A project was conducted in a factory in Northeast China to find an optimal composition so that the cement grouting material formed has the desired properties. The objectives were to minimize the coefficient of bleeding $BL$ (at water/cement ratio of 0.6) and maximize the compressive strength $R_{28}$ (at water/cement ratio 0.8) which is measured twenty-eight days after the cement mixture has set. Four controllable variables were considered: percentages of fly ash, silica fume, CMC and cement, denoted by $x_1$, $x_2$, $x_3$ and $x_4$ respectively. It was decided to perform an experiment and to use the result obtained to construct regression models for $BL$ and $R_{28}$ in order to locate the optimal points. Since the factory had tight production schedules, very little resources were available for this project, and altogether only about 20 runs could be performed in the experiment.

Experience showed that ranges of $x_1$, $x_2$, $x_3$ in percentages should lie within the following ranges:

$$5 \leq x_1 \leq 20, \quad 1 \leq x_2 \leq 2.4, \quad 0.3 \leq x_3 \leq 1.0; \quad (1)$$

otherwise, either $BL$ might be too large or $R_{28}$ might be too low. Because of the constraint $x_1 + x_2 + x_3 + x_4 = 100$; the amount of cement $x_4$ should therefore lie within the range:

$$76.6 \leq x_4 \leq 93.7 \quad (2)$$

and any design layout can be expressed in terms of the variables $x_1$, $x_2$, $x_3$. An experiment with such a setup is referred to as an experiment with mixtures. For a review and a monograph of experiments with mixtures, see Chan (2000) and Cornell (2002). The design space of this experiment, which is a subregion of the following simplex,

$$S_{100}^4 = \{ (x_1, x_2, x_3) : x_1, x_2, x_3 \geq 0, x_1 + x_2 + x_3 \leq 100 \}$$

is represented as the tetrahedron with vertices $O$, $A$, $B$, $C$ in Fig. 1.

Experience shows that even slight changes of $x_1$, $x_2$, $x_3$, especially $x_1$, might have large effects on $R_{28}$ and $BL$. In order to scrutinize the effects of these factors, designs of two or three levels were clearly insufficient. It was decided that the experiment should be performed with the following fine scales of division on $x_1$, $x_2$, $x_3$:

- $x_1$: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20;
- $x_2$: 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4;
- $x_3$: 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.

Since $x_1$, $x_2$, $x_3$ are constrained by Eq. (1), only a small part of $S_{100}^4$ is the admissible region for the experiment. In Fig. 1, the coordinates of $O$, $A$, $B$, and $C$ are $(0, 0, 0)$, $(100, 0, 0)$, $(0, 100, 0)$, and $(0, 0, 100)$, respectively. The five triangular planes $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$, $A_4B_4C_4$, and $A_5B_5C_5$, which are parallel to triangle $ABC$, correspond to $x_4 = 79.0$, 82.0, 85.0, 88.0 and 91.0, respectively. The admissible regions of $x_1$, $x_2$, and $x_3$ on these triangles are the parallelograms marked 1, 2, 3, 4, 5, respectively. The parallelogram marked $i$ lies on the plane of triangle $A_iB_iC_i$ ($i = 1, 2, 3, 4, 5$). The positions of the points $A_i$, $B_i$, $C_i$ ($i = 1, \ldots, 5$) in Fig. 1 are not to scale on the axes, nor are the positions and sizes of the five parallelograms.

2. UNIFORM DESIGN

Traditional designs such as fractional designs have been widely used in the industry (Dehnad, 1989; Montgomery, 2001). Designs of low levels (such as two or three levels) can be conveniently used to identify factors that have significant effects on the outcome. However, a restriction of such designs is their small number of levels on the factors, which
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Figure 1. The tetrahedron with vertices A, B, C, O represents the region $S^{4-1} = \{(x_1, x_2, x_3, x_4) : x_1 + x_2 + x_3 + x_4 = 1; x_i \geq 0 \ (i = 1, 2, 3, 4)}$.

does not provide reliable estimates of the slight changes on the response caused by fine variations in the design factors. If a larger number of levels is used, the experiment can be performed with better coverage of the design space but at the expense of a large number of runs. For example, the number of runs $n$ of an $L_n(s_1^r \times s_2^r)$ orthogonal array must be at least a multiple of $s_1^2 s_2^2$ when $r_1, r_2 \geq 2$. Even if $s_1$ and $s_2$ are of moderate sizes, say, $s_1 = 4$ and $s_2 = 5, s_1^2 s_2^2$ will be as large as 400, and such a number of runs usually cannot be realized in an actual industrial experiment.

The theoretical results in Fang and Mukerjee (2000) reveal that the success of factorial designs in exploration of the response surface is due to their uniformity in coverage of the whole design space, rather than their combinatorial or orthogonal property. If wide coverage of the design space is of primary importance, even when the number of levels of factors is large, designs with smaller numbers of runs can be constructed by sacrificing combinatorial properties of the design. A design without nice combinatorial properties can certainly serve the purpose of response surface analysis in industrial experiments, although such designs may not be desirable for ANOVA in some situations, for example, in surveys in social sciences research. The uniform design is such a design, which was first suggested in Wang and Fang (1981) and has been studied theoretically by many authors in the past several decades (Fang and Mukerjee, 2000; Hickernell, 1998, 1999; Liu and Hickernell, 2000; Wang and Fang, 1995). The uniform design can be easily adopted in practice, even if the number of levels of some factors is very large, say 20 or more. Readers are referred to Fang and Wang (1994) as a text reference; Fang et al. (2000) for a brief
3. THE EXPERIMENT

The present experiment was performed with 16 levels in \( x_1 \) and 8 levels in each of \( x_2 \) and \( x_3 \), as shown in Sec. 1. An orthogonal array for this setup requires at least \( 16 \times 8^3 = 156 \) runs, which is too large to be practical. A uniform design \( U_{16}(16 \times 8^3) \) with 16 runs was used. The setup and the observed responses are shown in the first 6 columns and the first 16 rows of numbers in Table 1. The column \( BL \) contains the observed values of the coefficient of bleeding (in %) before the grouting process. The column \( R_{28} \) contains the observed compressive strength (in MPa) of the mixture 28 days after the cement has set. In Table 1, the numbers in parentheses (1), (2), etc., are the numbers representing the levels of factors in the \( U_{16}(16 \times 8^3) \) uniform design table.

These data were fitted with the following Scheffé quadratic model (Scheffé, 1958):

\[
\eta_2(x) = \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \leq i < j \leq q} \beta_{ij} x_i x_j
\]

where \( x = (x_1, \ldots, x_q) \in S_{100}^{q-1} \), and \( x \) is under the constant sum constraint \( x_1 + \cdots + x_q = 100 \) (\( q = 4 \)). By the substitution \( x_q = 100 - x_1 - \cdots - x_{q-1} \), the model \( \eta_2(x) \) in Eq. (3) can be equivalently expressed in terms of \( q-1 \) regressor variables \( x_1, \ldots, x_{q-1} \) as follows:

\[
\eta_2(x) = \sum_{i=1}^{q-1} \beta_i x_i + \beta_{q}(100 - x_1 - \cdots - x_{q-1}) + \sum_{1 \leq i < j \leq q-1} \beta_{ij} x_i x_j
\]

\[
= a_0 + \sum_{i=1}^{q-1} a_i x_i + \sum_{1 \leq i < j \leq q-1} a_{ij} x_i x_j
\]

### Table 1. Results of the experiment.

<table>
<thead>
<tr>
<th>Run number</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( BL )</th>
<th>( R_{28} )</th>
<th>( [BL] )</th>
<th>( [R_{28}] )</th>
<th>( e_{BL} )</th>
<th>( e_{R_{28}} )</th>
</tr>
</thead>
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<td>1</td>
<td>5 (1)</td>
<td>2.2 (7)</td>
<td>0.9 (7)</td>
<td>9.92</td>
<td>27.65</td>
<td>9.96</td>
<td>27.46</td>
<td>0.40</td>
<td>-0.69</td>
</tr>
<tr>
<td>2</td>
<td>6 (2)</td>
<td>1.8 (5)</td>
<td>0.8 (6)</td>
<td>11.26</td>
<td>26.58</td>
<td>11.35</td>
<td>26.24</td>
<td>0.80</td>
<td>-1.29</td>
</tr>
<tr>
<td>3</td>
<td>7 (3)</td>
<td>1.4 (3)</td>
<td>0.6 (4)</td>
<td>14.40</td>
<td>24.82</td>
<td>14.18</td>
<td>25.41</td>
<td>-1.55</td>
<td>2.39</td>
</tr>
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<td>4</td>
<td>8 (4)</td>
<td>1.0 (1)</td>
<td>0.5 (3)</td>
<td>16.03</td>
<td>23.15</td>
<td>16.02</td>
<td>23.62</td>
<td>-0.06</td>
<td>2.04</td>
</tr>
<tr>
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<td>9 (5)</td>
<td>2.2 (7)</td>
<td>0.3 (1)</td>
<td>12.46</td>
<td>22.49</td>
<td>12.57</td>
<td>21.92</td>
<td>0.88</td>
<td>-2.52</td>
</tr>
<tr>
<td>6</td>
<td>10 (6)</td>
<td>1.8 (5)</td>
<td>1.0 (8)</td>
<td>4.85</td>
<td>26.06</td>
<td>4.87</td>
<td>26.01</td>
<td>0.41</td>
<td>-0.20</td>
</tr>
<tr>
<td>7</td>
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<td>1.4 (3)</td>
<td>0.9 (7)</td>
<td>6.80</td>
<td>25.19</td>
<td>6.67</td>
<td>25.75</td>
<td>-1.91</td>
<td>2.24</td>
</tr>
<tr>
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<td>12 (8)</td>
<td>1.0 (1)</td>
<td>0.7 (5)</td>
<td>9.95</td>
<td>28.02</td>
<td>10.21</td>
<td>26.45</td>
<td>2.61</td>
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<td>0.6 (4)</td>
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<td>28.43</td>
<td>6.81</td>
<td>29.89</td>
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<td>5.14</td>
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<td>0.4 (2)</td>
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<td>22.08</td>
<td>10.13</td>
<td>21.95</td>
<td>-2.03</td>
<td>-0.57</td>
</tr>
<tr>
<td>11</td>
<td>15 (11)</td>
<td>1.6 (4)</td>
<td>0.3 (1)</td>
<td>12.40</td>
<td>17.15</td>
<td>12.51</td>
<td>17.07</td>
<td>0.089</td>
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</tr>
<tr>
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<td>16 (12)</td>
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<td>1.0 (8)</td>
<td>3.03</td>
<td>24.18</td>
<td>2.97</td>
<td>24.64</td>
<td>-1.98</td>
<td>1.91</td>
</tr>
<tr>
<td>13</td>
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<td>0.8 (6)</td>
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<td>3.51</td>
<td>29.78</td>
<td>0.00</td>
<td>-2.54</td>
</tr>
<tr>
<td>14</td>
<td>18 (14)</td>
<td>2.0 (6)</td>
<td>0.7 (5)</td>
<td>5.78</td>
<td>26.44</td>
<td>5.86</td>
<td>26.44</td>
<td>1.38</td>
<td>-0.01</td>
</tr>
<tr>
<td>15</td>
<td>19 (15)</td>
<td>1.6 (4)</td>
<td>0.5 (3)</td>
<td>9.73</td>
<td>20.35</td>
<td>9.88</td>
<td>20.01</td>
<td>1.54</td>
<td>-1.63</td>
</tr>
<tr>
<td>16</td>
<td>20 (16)</td>
<td>1.2 (2)</td>
<td>0.4 (2)</td>
<td>12.83</td>
<td>15.63</td>
<td>12.68</td>
<td>16.10</td>
<td>-1.17</td>
<td>3.04</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1.0</td>
<td>0.4</td>
<td>19.62</td>
<td>20.6</td>
<td>20.31</td>
<td>19.83</td>
<td>3.52</td>
<td>-3.75</td>
</tr>
<tr>
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<td>10</td>
<td>1.6</td>
<td>0.5</td>
<td>11.23</td>
<td>28.1</td>
<td>12.17</td>
<td>26.69</td>
<td>8.37</td>
<td>-1.47</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>1.8</td>
<td>0.8</td>
<td>4.91</td>
<td>28.7</td>
<td>5.20</td>
<td>29.59</td>
<td>5.90</td>
<td>-2.32</td>
</tr>
<tr>
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<td>2.0</td>
<td>0.6</td>
<td>7.63</td>
<td>28.4</td>
<td>7.46</td>
<td>27.47</td>
<td>-2.23</td>
<td>-1.75</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>1.8</td>
<td>0.7</td>
<td>6.61</td>
<td>24.6</td>
<td>6.58</td>
<td>23.83</td>
<td>-0.45</td>
<td>-3.12</td>
</tr>
</tbody>
</table>
where the \( \alpha \)'s and \( \beta \)'s are related by a one-to-one transformation. See Cornell (2002). A regression model selection procedure in the software package SAS was used to fit the data for \( BL \) and \( R_{28} \) in Table 1 using the form of \( y_2(x) \) in Eq. (4). The results are expressed in the form of Eq. (3) as follows:

\[
BL = 3.3366x_1 - 341.32x_1 + 0.36548x_4 \\
+ 2.9980x_1x_3 - 0.045752x_1x_4 + 5.5116x_2x_3 \\
- 0.056074x_2x_4 + 3.3229x_3x_4
\]  

(5)

\[
R_{28} = -6.6942x_1 + 3.4349x_2 - 4824.1x_3 \\
- 0.58536x_1x_2 + 50.485x_1x_3 + 0.085773x_1x_4 \\
+ 61.285x_2x_3 + 48.526x_3x_4
\]  

(6)

Here the form in Eq. (3) was used, since it contains all the variables \( x_1, x_2, x_3, x_4 \) and is more convenient for the purpose of contour line plotting. In Eq. (5) and Eq. (6), all terms are significant at the 0.1 level. The values of \( R^2 \) and \( C(p) \) of the regression equation in Eq. (5) are 0.999983 and 6.15525 and those in Eq. (6) are 0.999263 and 6.08415, respectively. The analysis of variance is shown in Table 2.

Table 2a. The ANOVA for \( BL \) in (3.5).

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of square</th>
<th>Degree of freedom</th>
<th>Mean square</th>
<th>( F )-ratio</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1635.73</td>
<td>8</td>
<td>204.466</td>
<td>5168.85</td>
<td>1.69065 \times 10^{-12}</td>
</tr>
<tr>
<td>Error</td>
<td>0.276902</td>
<td>7</td>
<td>0.039574</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1636.01</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2b. The ANOVA for \( R_{28} \) in (3.6).

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of square</th>
<th>Degree of freedom</th>
<th>Mean square</th>
<th>( F )-ratio</th>
<th>Probability</th>
</tr>
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<tbody>
<tr>
<td>Regression</td>
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<td>8</td>
<td>1210.80</td>
<td>1187.09</td>
<td>2.9027 \times 10^{-10}</td>
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<tr>
<td>Error</td>
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<td>7</td>
<td>1.01997</td>
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</tr>
<tr>
<td>Total</td>
<td>9693.55</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. BEHAVIOR OF \( BL \) AND \( R_{28} \)

In order to understand how \( BL \) and \( R_{28} \) change as \( x \) changes, contour lines are plotted on parts of the triangles \( A_1B_1C_1, \ldots, A_5B_5C_5 \) containing the admissible regions, which are the parallelograms marked 1, \ldots, 5 in Fig. 1. Figures 2(a) through (e) show the contour plots of \( BL \), and Figs. 3(a) through (e) show those of \( R_{28} \).

Putting \( x_4 = 100 - x_1 - x_2 - x_3 \), \( BL \) in Eq. (5) will be represented as:

\[
BL = 36.548 - 1.604x_1 + 0.045752x_1^2 - 5.97288x_2 \\
+ 0.101826x_1x_2 + 0.056074x_2^2 - 9.3954x_3 \\
- 0.279148x_1x_3 + 2.24477x_2x_3 - 3.3229x_3^2
\]  

(7)

where \( x_1, x_2, x_3 \) are free to vary in the region \( \{(x_1,x_2,x_3): 0 \leq x_1 \leq 20, 1 \leq x_2 \leq 2.4, 0.3 \leq x_3 \leq 1.0\} \). Since it follows from Eq. (7) that \( \partial BL/\partial x_2 = -5.97288 + 0.101826x_1 + 0.112148x_2 + 2.24477x_3 \leq -5.97288 + 0.101826 \times 20 + 0.112148 \times 2.4 + 2.24477 \times 1 < 0 \), and \( \partial BL/\partial x_3 = -9.3954 - 0.279148x_1 + 2.24477x_2 - 6.6458x_3 < -9.3954 - 0.279148 \times 1 + 2.24477 \times 2.4 - 6.6458 \times 0.3 < 0 \) in the region \( \{(x_1,x_2,x_3): 0 \leq x_1 \leq 20, 1 \leq x_2 \leq 2.4, 0.3 \leq x_3 \leq 1.0\} \), the minimum value of \( BL \) in this region is attained when \( x_2 = 2.4, x_3 = 1.0 \). Setting \( x_2 = 2.4 \) and \( x_3 = 1.0 \) in Eq. (7) and minimizing \( BL \) shows that \( BL \) has a minimum value 0.5292 at \( x_1 = 17.910 \). Hence, up to two decimal places in the design region constrained by Eqs. (1) and (2), \( \min(\{BL\}) = 0.52 \) is attained at \( (x_1,x_2,x_3) = (17.9,2.4,1.0) \). The contour plots in Figs. 2(a) through (d) show how \( BL \), represented by Eq. (7), changes as the variables as \( x_1, \ldots, x_4 \) change. These figures show that when...
Figure 2. (a) Counter plots of $BL$ on triangle $A_1B_1C_1$ ($x_4 = 79.0$); (b) counter plots of $BL$ on triangle $A_2B_2C_2$ ($x_4 = 82.0$); (c) counter plots of $BL$ on triangle $A_3B_3C_3$ ($x_4 = 85.0$); (d) counter plots of $BL$ on triangle $A_4B_4C_4$ ($x_4 = 88.0$); (e) counter plots of $BL$ on triangle $A_5B_5C_5$ ($x_4 = 91.0$).
x_4 = 79, 82, 85, 88, and 91, the minimum of BL is on the boundary of the design region where x_2 = 2.4 and x_3 = 1.0 (indicated by small circles in the figures), and this minimum increases as x_4 increases.

Putting x_4 = 100 - x_1 - x_2 - x_3, R_{28} in Eq. (6) will be represented as:

\[
\begin{align*}
R_{28} &= 1.8831x_1 - 0.085773x_1^2 + 3.4349x_2 \\
& \quad - 0.671133x_1x_2 + 24.491x_3 + 1.87323x_1x_3 \\
& \quad + 12.759x_2x_3 - 48.526x_3^2
\end{align*}
\]

(8)

where x_1, x_2, x_3 are free to vary in the region \{(x_1, x_2, x_3): 5 \leq x_1 \leq 20, 1 \leq x_2 \leq 2.4, 0.3 \leq x_3 \leq 1.0\}. From Eq. (8), putting \(\partial R_{28}/\partial x_1 = \partial R_{28}/\partial x_2 = \partial R_{28}/\partial x_3 = 0\) gives \((x_1, x_2, x_3) = (16.6327, 0.244961, 0.605678)\), which is outside the admissible region \{(x_1, x_2, x_3): 5 \leq x_1 \leq 20, 1 \leq x_2 \leq 2.4, 0.3 \leq x_3 \leq 1.0\}. Hence, the maximum of R_{28} in this region must be attained on the boundary of the region. Finding the maximums of R_{28} on each of the faces, edges, and vertices of this region shows that the global maximum of R_{28} in this region is 33.6301, which is attained on the face \(x_2 = 2.4\) at \((x_1, x_3) = (10.4302, 0.81068)\). Hence, up to two decimal places in the design region constrained by Eqs. (1) and (2), \(\max(R_{28}) = 33.63\) is attained at \((x_1, x_2, x_3) = (10.43, 2.40, 0.81)\). The contour plots in Figs. 3(a) through (d) show how R_{28} represented by Eq. (8) changes as the variables vary.
Figure 3. (a) Counter plots of $R_{28}$ on triangle $A_1B_1C_1 (x_4 = 79.0)$; (b) counter plots of $R_{28}$ on triangle $A_2B_2C_2 (x_4 = 82.0)$; (c) counter plots of $R_{28}$ on triangle $A_3B_3C_3 (x_4 = 85.0)$; (d) counter plots of $R_{28}$ on triangle $A_4B_4C_4 (x_4 = 88.0)$; (e) counter plots of $R_{28}$ on triangle $A_5B_5C_5 (x_4 = 91.0)$. 

Note: The values in the figure correspond to the coordinates of the vertices of the triangles, which are used to plot the counter plots of $R_{28}$.
$x_1, \ldots, x_4$ change. These figures show that when $x_4 = 79, 82, 85, 88, \text{ and } 91$, the maximums of $R_{28}$ occur when $x_2 = 2.4$ (indicated by small circles in the figures).

5. OPTIMIZATION

Analysis in the last section shows that in the admission region constrained by Eqs. (1) and (2), $BL$ has a predicted minimum of 0.52 at $(x_1, x_2, x_3) = (17.9, 2.4, 1.0)$, while $R_{28}$ has a predicted maximum of 33.63 at $(x_1, x_2, x_3) = (10.43, 2.4, 0.81)$, which is different from the minimum point of $BL$. As a trade-off, optimal compositions of the cement grouting mixture can be obtained by considering the material cost as well as the magnitudes of $BL$ and $R_{28}$.

The material costs of fly ash, silica fume, CMC, and cement are $500, $4000, $5000, and $320 per tonne (prices have been scaled here for confidentiality reason). The material cost per tonne for a mixture of composition $x = (x_1, x_2, x_3, x_4)'$ is:

$$
\text{cost} = 5x_1 + 40x_2 + 50x_3 + 3.2x_4 \\
= 5(100 - x_2 - x_3 - x_4) + 40x_2 + 50x_3 + 3.2x_4
$$

The last equality follows from the constraint $x_1 + x_2 + x_3 + x_4 = 100$. Figures 3(a) through (e) show that within the range of experimentation, setting the
amounts of silica fume and CMC at their maximums \((x_2 = 2.4, x_3 = 1.0)\) gives the best values of \(B_L\), while setting \(x_2\) at the maximum value 2.4 and setting \(x_3 = 0.8\) (near the maximum value 1.0) yields the best value of \(R_{28}\). However, these two additives are the most expensive among all ingredients. Optimal combinations of the amounts \(x_1, x_2, x_3, x_4\) of the four ingredients—fly ash, silica fume, CMC, and cement—can be obtained by minimizing the cost in Eq. (4) under some conditions on \(B_L\) and \(R_{28}\), say \(B_L \leq 1, 2, 3\) and \(R_{28} \geq 30, 25, 20\). This optimization can be performed with the aid of software such as MATLAB or Mathematica or done graphically as follows.

1. Substitute \(x_1\) in Eq. (5) by \(x_1 = 100 - x_2 - x_3 - x_4\), simplify the result, and express \(x_2\) in terms of \(x_3, x_4\), and \(B_L\):

\[
x_2 = (-333.66 + BL + 44.8566x_3 + 2.998x_3^2
\]

\[
+ 7.54632x_4 - 0.370752x_3x_4 - 0.045752x_3^2)
\]

\[
\div (-3.3366 + 2.5136x_3 - 0.010322x_4) \quad (10)
\]

2. Substitute \(x_2\) in Eq. (10) into Eq. (9) to express cost in terms of \(x_3, x_4\), and \(B_L\).

3. Substitute \(x_2\) in Eq. (10) into Eq. (6) to express \(R_{28}\) in terms of \(x_3, x_4\), and \(B_L\).

4. Use the result obtained in step 2 to plot contour maps of cost on the \(x_3x_4\)-plane for \(B_L = 1, 2, 3, 4\). These are shown in Figs. 4(a) through (d).

5. Use the result obtained in step 3 to obtain contour lines of \(B_L = 20, 25, 30\) on the \(x_3x_4\)-plane and plot these contour line on

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{(a) Counter plots of cost for \(B_L = 1\); (b) counter plots of cost for \(B_L = 2\); (c) counter plots of cost for \(B_L = 3\); (d) counter plots of cost for \(B_L = 4\).}
\end{figure}
the contour maps of cost in Figs. 4(a) through 4(d). Locate the points on these contour maps with minimum values of cost that correspond to $R_{28} = 20, 25, 30$. These minimum points are indicated by small circles in Fig. 4(a) through 4(d). The results are shown in Table 3.

From Table 3, the following three grades of cements grouting material can be suggested:

**Grade I:** \((x_1, x_2, x_3, x_4) = (16.29, 2.21, 1.00, 80.50), BL = 1.00, R_{28} = 30.007, cost = $477.31\) (per tonne).

**Grade II:** \((x_1, x_2, x_3, x_4) = (17.87, 1.17, 0.99, 81.30), BL = 3.00, R_{28} = 25.001, cost = $440.46\) (per tonne).

**Grade III:** \((x_1, x_2, x_3, x_4) = (19.32, 0.30, 1.00, 79.38), BL = 4.00, R_{28} = 21.499, cost = $412.62\) (per tonne).

The contour plots in the figures suggest that some other values of the compositions \((x_1, x_2, x_3, x_4)\) could give better values of \(BL\) and \(R_{28}\) than those of grade I mixture. As seen from Figs. 2(a) through 3(e), further decrease of \(BL\) and increase of \(R_{28}\) will force \(x_2\) and \(x_3\) to increase beyond their experimental limits of \(x_2 \leq 2.4\) and \(x_3 \leq 1.0\), respectively. However, extrapolating the empirical expression Eq. (7) for \(x_2 > 2.4\) and \(x_3 > 1.0\) will result in negative values of \(BL\), which is physically impossible. This suggests that further experiments can be done to investigate how the cement mixture behaves when \(x_2 > 2.4\) and \(x_3 > 1.0\).

By adopting segmented empirical models that take...
the zero value (not negative value) over a region in
the simplex and take positive values elsewhere
in the simplex, a cement grouting material with
$BL = 0$ and $R_{28} \gg 30$ may be obtained. Such a
grouting mixture can be expected to contain higher
proportions of silica fume and CMC and hence be
more costly.

6. CONCLUSION

Factorial designs and orthogonal arrays have
been widely used in design of industrial experiments.
When the number of factors is large or the numbers
of levels of the factors are large, these designs require
a large number of runs, which may not be possible to
achieve in practice because of various constraints. In
such a case, the uniform design is an excellent
alternative that can be used for the experiments.

The uniform design has been studied extensively by
mathematicians and statisticians for more than two
decades, but its application in industries worldwide
still has to be promoted. However, there have already
been many successful applications of uniform designs
in industry, especially in petroleum engineering,
quality engineering, and system engineering. See
Uniform Design Association of China (Selected
Papers in Uniform Design, 1997) for a collection of
published works.

This article gives a case study to illustrate how a
uniform design is used in an experiment with 16 runs
and 3 variables each having 16, 8, and 8 levels,
respectively, in an experiment for cement mixture
formation. In this case study, we provided a complete
procedure for applying uniform design in product
formation in industry—from design of experiment, to
data analysis, to optimization. This can be easily
extended and applied to other situations.
Table 3. Minimum costs that correspond to $BL = 1, 2, 3, 4$ and $R_{28}/C_{21} = 30, 25, 20$.

<table>
<thead>
<tr>
<th>$BL$</th>
<th>Region of optimization</th>
<th>Optimal point $(x_1, x_2, x_3, x_4)$</th>
<th>Value of $R_{28}$</th>
<th>Value of cost (minimum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>$R_{28} \geq 30$</td>
<td>$A = (16.29, 2.21, 1.00, 80.50)$</td>
<td>30.007</td>
<td>477.31</td>
</tr>
<tr>
<td>1.00</td>
<td>$R_{28} \geq 25$</td>
<td>$B = (17.28, 2.14, 1.00, 79.58)$</td>
<td>29.105</td>
<td>476.51</td>
</tr>
<tr>
<td>1.00</td>
<td>$R_{28} \geq 20$</td>
<td>$B = (17.28, 2.14, 1.00, 79.58)$</td>
<td>29.105</td>
<td>476.51</td>
</tr>
<tr>
<td>2.00</td>
<td>$R_{28} \geq 30$</td>
<td>$C = (14.12, 2.08, 0.99, 82.81)$</td>
<td>30.003</td>
<td>468.12</td>
</tr>
<tr>
<td>2.00</td>
<td>$R_{28} \geq 25$</td>
<td>$D = (17.96, 1.52, 1.00, 79.52)$</td>
<td>26.083</td>
<td>455.22</td>
</tr>
<tr>
<td>2.00</td>
<td>$R_{28} \geq 20$</td>
<td>$D = (17.96, 1.52, 1.00, 79.52)$</td>
<td>26.083</td>
<td>455.22</td>
</tr>
<tr>
<td>3.00</td>
<td>$R_{28} \geq 30$</td>
<td>$E = (14.08, 1.97, 0.94, 83.01)$</td>
<td>30.002</td>
<td>461.88</td>
</tr>
<tr>
<td>3.00</td>
<td>$R_{28} \geq 25$</td>
<td>$F = (16.50, 1.21, 0.99, 81.30)$</td>
<td>25.001</td>
<td>440.46</td>
</tr>
<tr>
<td>3.00</td>
<td>$R_{28} \geq 20$</td>
<td>$F = (16.50, 1.21, 0.99, 81.30)$</td>
<td>25.001</td>
<td>440.46</td>
</tr>
<tr>
<td>4.00</td>
<td>$R_{28} \geq 30$</td>
<td>$G = (18.64, 0.91, 1.00, 79.45)$</td>
<td>23.551</td>
<td>433.92</td>
</tr>
<tr>
<td>4.00</td>
<td>$R_{28} \geq 25$</td>
<td>$G = (18.64, 0.91, 1.00, 79.45)$</td>
<td>23.551</td>
<td>433.92</td>
</tr>
<tr>
<td>4.00</td>
<td>$R_{28} \geq 20$</td>
<td>$G = (18.64, 0.91, 1.00, 79.45)$</td>
<td>23.551</td>
<td>433.92</td>
</tr>
</tbody>
</table>

Figure 4. Continued.
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REFERENCES


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