International Journal of Reliability, Quality and Safety Engineering
Vol. 8, No. 2 (2001) 159–171

PERFORMANCE OF SENSITIZING RULES ON SHEWHART CONTROL CHARTS WITH AUTOCORRELATED DATA

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Received 19 December 2000
Revised 9 April 2001

Sensitizing Rules are commonly applied to Shewhart Charts to increase their effectiveness in detecting shifts in the mean that may otherwise go unnoticed by the usual "out-of-control" signals. The purpose of this paper is to demonstrate how well these rules actually perform when the data exhibit autocorrelation compared to non-correlated data. Since most control chart data are collected as time series, it is of interest to examine the performance of Shewhart's $\bar{x}$ Chart using data generated from typical time series models. In this paper, measurements arising from autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) processes are examined using Shewhart Control Charts in conjunction with several sensitizing rules. The results indicate that the rules work well when there are strong autocorrelative relationships, but are not as effective in recognizing small to moderate levels of correlation. We conclude with the recommendation to practitioners that they use a more definitive measure of autocorrelation such as the Sample Autocorrelation Function correlogram to detect dependency.

Keywords: Autoregressive; Moving Average; Runs Tests; Shewhart Control Charts; Statistical Process Control; Time Series.

1. Introduction

The standard analysis and interpretation of a Shewhart $\bar{x}$ Chart assumes that the data are normally and independently distributed (NID) with mean μ and standard deviation σ which remain constant over time. It is common to apply runs tests in the analysis to increase the chart's effectiveness in detecting small shifts in the process. Such tests are referred to as *sensitizing rules*[9] and are widely used in practice as they are easy to apply.[6] Some of these tests are found in Table 1.

The sensitizing rules make use of exclusive and exhaustive zones which divide the area between the upper and lower control limits into three regions. The zones

Table 1. Some sensitizing rules for Shewhart Control Charts.

Rule 1	A point falls outside the 3 sigma limit
Rule 2	8 points in a row in zone C or beyond on the same side of the center line
Rule 3	6 points in a row increasing or decreasing
Rule 4	14 points in a row alternating up and down
Rule 5	2 out of 3 points in a row in zone A or beyond on the same side of the center line
Rule 6	4 out of 5 points in a row in zone B or beyond on the same side of the center line
Rule 7	15 points in a row in zone C
Rule 8	8 points in a row not in zone C

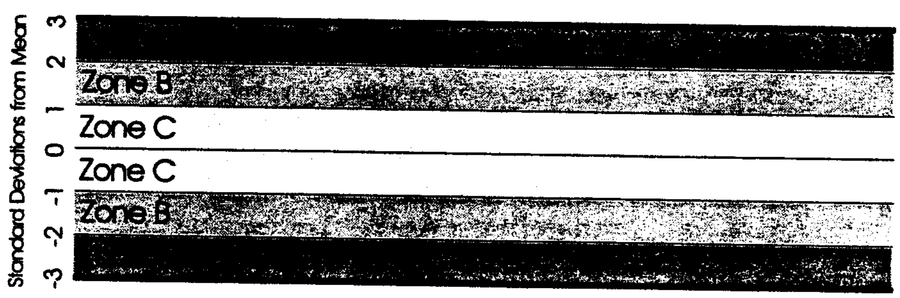

Fig. 1. Shewhart Chart with zones.

refer to the region between the center line and the ±1 sigma limits as zone C; between the ±1 sigma limits and ±2 sigma limits as zone B; and between the ±2 and ±3 sigma limits as zone A. Figure 1 displays the zones graphically. Using these rules increases the chance of detecting changes in the process mean, but may lead to a greater *Type I* error rate.

Since the data for Shewhart's $\bar{x}$ Chart are collected as a time series, we show how sensitizing rules identify a violation of the independency assumption by simulating linearly autocorrelated data generated from conventional time series models. This paper describes the autocorrelation structures which are used in the simulation demonstrating the sensitizing rules and provides an interpretation of the results of the simulation followed by a study of the impact of series length on the probability of false positives. We conclude with a discussion of the outcomes and recommendations for practitioners.

2. Autocorrelated Data

The standard assumptions associated with the use of control charts include the data being generated by an NID (μ, σ) process with the parameters fixed but unknown.[6] This assumption is often invalid as time series data is frequently correlated. When a series drifts over time, it is said to be autocorrelated. The level of autocorrelation is measured using the autocorrelation function:

$$\rho_k = \frac{\mathrm{Cov}(x_t, x_{t-k})}{\mathrm{Var}(x_t)}, \quad k = 0, 1, \ldots$$

and estimated using:

$$r_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t-k} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}, \quad k = 0, 1, \ldots, K$$

where N is the length of the time series. As a general rule, the first $K \leq N/4$ samples are computed.[7]

In this study, autocorrelated data are simulated using *Linear Gaussian Models* as the generating process. Linear Gaussian Models are frequently used in time series analysis to explain the movement of a series as a function of its past performance plus random shocks. We will use the Linear Gaussian Models described below to induce correlation in the data.

The first type of linear model studied will be the *autoregressive* process of order $p\,(\mathrm{AR}(p))$ that is characterized by

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \epsilon_t\,.$$

The AR(p) is a weighted average of the past performance with weights ϕ_i and a normal error term $\epsilon_t \sim N(0, \sigma^2)$. Such a model is used when the change in the series at any point in time is linearly correlated with previous changes.

A second type of linear model that will be used in the analysis is the *moving average* process of order $q\,(\mathrm{MA}(q))$ that is characterized by

$$Y_t = \mu + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q}\,.$$

The MA(q) is a weighted average (with weights θ_i) of random shocks (i.e., ϵ_i) spanning q periods. Each of the ϵ_i's is assumed to follow a normal distribution with mean 0 and standard deviation σ. A moving average model is used when there is a linear dependence on past performance. It is interesting to note that the system has a q-period memory meaning that a random shock persists for exactly q periods.

Combining the two models above results in the *mixed autoregressive-moving average* (ARMA(p, q)) process characterized by

$$Y_t = c + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \epsilon_t - \theta_1\epsilon_{t-1} - \cdots - \theta_q\epsilon_{t-q}\,.$$

This type of scheme is used when both moving average and autoregressive tendencies are present.

3. Simulation Procedure

Our goal is to evaluate the ability of the sensitizing rules to detect dependency in a series of observations, not to decide on an optimal batch size. Thus, we will only look at series of individual observations (batch size of 1). For each model, a series of 100 data points was generated with Normal (0, 1) error terms. The NID case occurs when all parameter values of the AR, MA or ARMA model are set to zero and will serve as a "benchmark" for comparison. In order to cover most typical cases, we chose to use −0.9, −0.5, 0.1, 0.5, and 0.9 and their pairwise combinations as values for ϕ_i and θ_i in the models described in Sec. 2 to simulate time series. Shewhart Control limits are then determined using the mean of the series as the center line and the moving range of successive observations to determine the control limits. The moving range is defined as $\text{MR}_i = |x_i - x_{i-1}|$.[6] The mean of the moving range is used to estimate the process variability. The interpretation of the chart is then similar to that of the ordinary Shewhart-$\bar{x}$ Control Chart.

All eight sensitizing rules were then performed on the control chart noting when each rule was violated. Ten thousand (10,000) sets of 100 data points were generated via this process for the different linear models. The values reported are the fraction of generated series found in violation of each rule and the percentage of series which violated at least one of the rules. The series were generated and tested using the statistical software package *S-plus*.

4. Results and Discussion

Tables 2 through 4 show the results from the simulations. In the following section we study the results of each model simulation, examining each rule and its performance under the various models.

Rule 1: A point falls outside the 3 sigma limit

Rule 1 corresponds to having an observation fall relatively far from the process mean. Violation of this rule can indicate an out of control point or dependency of the process. This rule is typically violated when the generating process has a large autoregressive coefficient in absolute value or negatively large moving average term. For example, AR(1)-6, AR(2)-25, MA(2)-1 and ARMA-21 are all examples of models detected by this rule. However, models such as AR(1)-1, AR(2)-2, MA(1)-6, MA(2)-16 and ARMA(1,1)-4 are not detected by this rule, as can be seen in Tables 2, 3, and 4.

Rule 2: 8 points in a row in zone C or beyond on the same side of the center line

Rule 2 corresponds to a trend in the data. Violation of this rule is indicative of dependency in the data. This rule is typically violated when ϕ_2 is large for the AR schemes, when θ_1 and θ_2 are negatively large for the MA schemes and when ϕ_1 is large and θ_1 is negatively large for the ARMA scheme. Models AR(1)-6, AR(2)-20, MA(2)-1 and ARMA-21 are examples where this rule is effective.

Rule 3: 6 points in a row increasing or decreasing

Rule 3 also corresponds to a trend in the data. Violation of this rule is indicative of positive autocorrelation in the data. It is typically violated by AR(2) schemes when

Table 2. Results of the AR simulations. Numbers indicate the fraction of times the rule was violated.

Case	ϕ_1	ϕ_2	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	% Violated
NID	0.0	0.0	0.2365	0.2781	0.0330	0.0856	0.1724	0.2636	0.0773	0.0060	68.70
AR(1)-1	−0.9	0.0	0.0019	0.0049	0.0012	0.9933	0.0431	0.0000	0.9951	0.3441	99.95
AR(1)-2	−0.5	0.0	0.0238	0.0419	0.0051	0.4778	0.0318	0.0025	0.5774	0.0076	78.18
AR(1)-3	−0.1	0.0	0.1473	0.1940	0.0247	0.1238	0.0874	0.1413	0.1216	0.0051	58.07
AR(1)-4	0.1	0.0	0.3480	0.3899	0.0429	0.0634	0.3164	0.4455	0.0448	0.0127	82.04
AR(1)-5	0.5	0.0	0.9374	0.8952	0.1967	0.0195	0.9725	0.9859	0.0071	0.2393	99.95
AR(1)-6	0.9	0	1.0000	1.0000	0.6709	0.0063	1.0000	1.0000	0.0007	0.9912	100.00
AR(2)-1	−0.9	−0.9	0.0090	0.0005	0.0003	0.0031	0.0000	0.0223	0.9388	0.0098	93.94
AR(2)-2	−0.9	−0.5	0.0101	0.0021	0.0012	0.1368	0.0041	0.0003	0.7731	0.0007	80.50
AR(2)-3	−0.9	0.1	0.0008	0.0041	0.0005	0.9995	0.0213	0.0000	1.0000	0.4600	100.00
AR(2)-4	−0.9	0.5	1.0000	1.0000	0.0003	1.0000	1.0000	0.0000	1.0000	1.0000	100.00
AR(2)-5	−0.9	0.9	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	100.00
AR(2)-6	−0.5	−0.9	0.0233	0.0006	0.0004	0.0009	0.0016	0.0053	0.8513	0.0041	85.44
AR(2)-7	−0.5	−0.5	0.0429	0.0103	0.0027	0.0238	0.0014	0.0118	0.4507	0.0027	49.36
AR(2)-8	−0.5	0.1	0.0181	0.0538	0.0071	0.6867	0.0461	0.0017	0.6819	0.0210	90.00
AR(2)-9	−0.5	0.5	0.0004	0.0397	0.0028	0.9995	0.0257	0.0000	0.9973	0.3839	100.00
AR(2)-10	−0.5	0.9	1.0000	1.0000	0.0010	1.0000	1.0000	0.0000	1.0000	1.0000	100.00
AR(2)-11	0.1	−0.9	0.1129	0.0073	0.0030	0.0001	0.0850	0.0004	0.6099	0.3413	75.14
AR(2)-12	0.1	−0.5	0.2831	0.0702	0.0162	0.0028	0.1681	0.0518	0.1171	0.0366	52.82
AR(2)-13	0.1	0.1	0.3517	0.4929	0.0527	0.1091	0.3847	0.5506	0.0505	0.0140	88.28
AR(2)-14	0.1	0.5	0.4373	0.8915	0.0754	0.5482	0.7049	0.8292	0.0842	0.0872	99.26
AR(2)-15	0.1	0.9	0.6598	0.9938	0.0718	0.9815	0.9142	0.9189	0.2489	0.6733	100.00
AR(2)-16	0.5	−0.9	0.3881	0.0247	0.0113	0.0000	0.5116	0.0005	0.3975	0.0612	75.28
AR(2)-17	0.5	−0.5	0.7355	0.2553	0.0741	0.0010	0.7920	0.3686	0.0320	0.0874	95.42
AR(2)-18	0.5	0.1	0.9643	0.9606	0.2186	0.0320	0.9885	0.9949	0.0053	0.3812	99.98

Table 2. (*Continued*)

Case	ϕ_1	ϕ_2	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	% Violated
AR(2)-19	0.5	0.5	0.0522	0.9999	0.3034	0.1974	0.5562	0.9954	0.8527	0.8557	100.00
AR(2)-20	0.5	0.9	1.0000	1.0000	1.0000	0.0333	1.0000	1.0000	0.0000	1.0000	100.00
AR(2)-21	0.9	−0.9	0.8234	0.0985	0.0651	0.0000	0.9631	0.0399	0.1979	0.3501	98.72
AR(2)-22	0.9	−0.5	0.9968	0.7306	0.3712	0.0005	0.9999	0.9772	0.0073	0.3415	100.00
AR(2)-23	0.9	0.1	0.0398	1.0000	0.7055	0.0117	0.5593	0.9961	0.9280	0.9413	100.00
AR(2)-24	0.9	0.5	1.0000	1.0000	1.0000	0.0006	1.0000	1.0000	0.0000	1.0000	100.00
AR(2)-25	0.9	0.9	1.0000	1.0000	1.0000	0.0005	1.0000	1.0000	0.0000	1.0000	100.00

Table 3. Results of the MA simulations. Numbers indicate the fraction of times the rule was violated.

Case	θ_1	θ_2	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	% Violated
NID	0.0	0.0	0.2365	0.2781	0.0330	0.0856	0.1724	0.2636	0.0773	0.0060	68.70
MA(1)-1	−0.9	0.0	0.9534	0.7745	0.2245	0.0000	0.9793	0.9516	0.0107	0.1935	99.94
MA(1)-2	−0.5	0.0	0.8568	0.6905	0.1384	0.0027	0.8949	0.8921	0.0125	0.1071	99.49
MA(1)-3	−0.1	0.0	0.3496	0.3755	0.0447	0.0598	0.3211	0.4443	0.0475	0.0136	81.73
MA(1)-4	0.1	0.0	0.1511	0.1938	0.0220	0.1163	0.0859	0.1368	0.1194	0.0035	56.92
MA(1)-5	0.5	0.0	0.0387	0.0143	0.0039	0.2077	0.0119	0.0023	0.4200	0.0020	56.77
MA(1)-6	0.9	0.0	0.0262	0.0010	0.0030	0.2317	0.0067	0.0006	0.5823	0.0024	68.39
MA(2)-1	−0.9	−0.9	0.9979	0.9504	0.4658	0.0794	0.9996	0.9987	0.0053	0.5189	100.00
MA(2)-2	−0.9	−0.5	0.9986	0.9406	0.4271	0.0066	1.0000	0.9981	0.0033	0.5102	100.00
MA(2)-3	−0.9	0.1	0.9046	0.7049	0.1813	0.0000	0.9444	0.9058	0.0122	0.1334	99.75
MA(2)-4	−0.9	0.5	0.5333	0.3350	0.0701	0.0000	0.5102	0.4660	0.0379	0.0321	88.35
MA(2)-5	−0.9	0.9	0.2618	0.0848	0.0286	0.0018	0.1427	0.0948	0.0902	0.0135	51.30
MA(2)-6	−0.5	−0.9	0.8777	0.9099	0.2119	0.3094	0.9716	0.9776	0.0192	0.2121	99.91

Table 3. (*Continued*)

Case	θ_1	θ_2	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	% Violated
MA(2)-7	−0.5	−0.5	0.9402	0.8943	0.2320	0.1457	0.9806	0.9869	0.0092	0.2438	99.99
MA(2)-8	−0.5	0.1	0.7806	0.5949	0.1118	0.0006	0.8251	0.8120	0.0133	0.0745	98.66
MA(2)-9	−0.5	0.5	0.4397	0.2074	0.0505	0.0000	0.3686	0.2799	0.0550	0.0280	77.48
MA(2)-10	−0.5	0.9	0.2368	0.0356	0.0223	0.0002	0.1280	0.0503	0.1158	0.0143	45.30
MA(2)-11	0.1	−0.9	0.1445	0.6174	0.0547	0.5254	0.3871	0.3892	0.2572	0.0132	94.70
MA(2)-12	0.1	−0.5	0.1385	0.5167	0.0440	0.4304	0.2890	0.3207	0.2090	0.0078	89.52
MA(2)-13	0.1	0.1	0.1539	0.1305	0.0214	0.0678	0.0726	0.1044	0.1186	0.0038	49.39
MA(2)-14	0.1	0.5	0.1843	0.0228	0.0194	0.0021	0.0727	0.0309	0.1393	0.0110	38.34
MA(2)-15	0.1	0.9	0.2139	0.0122	0.0194	0.0001	0.0954	0.0230	0.1542	0.0173	41.83
MA(2)-16	0.5	−0.9	0.0261	0.2677	0.0174	0.5401	0.1003	0.0264	0.6133	0.0068	88.38
MA(2)-17	0.5	−0.5	0.0259	0.1446	0.0070	0.4581	0.0569	0.0073	0.6174	0.0060	83.22
MA(2)-18	0.5	0.1	0.0447	0.0082	0.0062	0.1515	0.0077	0.0028	0.3701	0.0006	49.34
MA(2)-19	0.5	0.5	0.1029	0.0039	0.0087	0.0251	0.0162	0.0071	0.2189	0.0059	33.83
MA(2)-20	0.5	0.9	0.1927	0.0167	0.0184	0.0005	0.0741	0.0244	0.1398	0.0123	38.51
MA(2)-21	0.9	−0.9	0.0099	0.0549	0.0021	0.5103	0.0303	0.0005	0.8369	0.0082	92.07
MA(2)-22	0.9	−0.5	0.0102	0.0107	0.0005	0.4306	0.0158	0.0002	0.8187	0.0056	89.01
MA(2)-23	0.9	0.1	0.0332	0.0007	0.0033	0.1915	0.0062	0.0003	0.5008	0.0011	60.53
MA(2)-24	0.9	0.5	0.0860	0.0091	0.0095	0.0531	0.0104	0.0077	0.2369	0.0028	36.20
MA(2)-25	0.9	0.9	0.1935	0.0400	0.0185	0.0056	0.0783	0.0463	0.1253	0.0090	40.86

Table 4. Results of the ARMA(1, 1) simulations. Numbers indicate the fraction of times the rule was violated.

Case	ϕ_1	θ_1	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	% Violated
NID	0.0	0.0	0.2365	0.2781	0.0330	0.0856	0.1724	0.2636	0.0773	0.0060	68.70
ARMA(1, 1)-1	−0.9	−0.9	0.2187	0.2644	0.0303	0.1414	0.1756	0.2416	0.1107	0.0083	69.93

Table 4. (*Continued*)

Case	ϕ_1	θ_1	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	% Violated
ARMA(1, 1)-2	−0.9	−0.5	0.0109	0.0711	0.9453	0.0074	0.9453	0.0512	0.0031	0.8121	97.81
ARMA(1, 1)-3	−0.9	0.1	0.0010	0.0024	0.0007	0.9940	0.0392	0.0000	0.9983	0.3623	100.00
ARMA(1, 1)-4	−0.9	0.5	0.0004	0.0001	0.0003	0.9967	0.0353	0.0000	0.9994	0.4365	100.00
ARMA(1, 1)-5	−0.9	0.9	0.0005	0.0000	0.0001	0.9959	0.0361	0.0000	0.9990	0.4480	99.99
ARMA(1, 1)-6	−0.5	−0.9	0.5754	0.4672	0.0710	0.0000	0.5686	0.6111	0.0280	0.0317	92.25
ARMA(1, 1)-7	−0.5	−0.5	0.2398	0.2759	0.0357	0.0903	0.1682	0.2730	0.0765	0.0052	68.84
ARMA(1, 1)-8	−0.5	0.1	0.0173	0.0209	0.0041	0.5230	0.0286	0.0005	0.6696	0.0091	83.82
ARMA(1, 1)-9	−0.5	0.5	0.0085	0.0014	0.0010	0.6155	0.0231	0.0001	0.8823	0.0192	94.72
ARMA(1, 1)-10	−0.5	0.9	0.0074	0.0001	0.0007	0.6335	0.0213	0.0000	0.9148	0.0218	96.44
ARMA(1, 1)-11	0.1	−0.9	0.9820	0.8401	0.2830	0.0000	0.9945	0.9782	0.0057	0.2640	99.98
ARMA(1, 1)-12	0.1	−0.5	0.9260	0.7758	0.1779	0.0017	0.9615	0.9451	0.0056	0.1594	99.92
ARMA(1, 1)-13	0.1	0.1	0.2327	0.2820	0.0299	0.0888	0.1716	0.2675	0.0730	0.0068	68.37
ARMA(1, 1)-14	0.1	0.5	0.0517	0.0276	0.0066	0.1727	0.0134	0.0076	0.3252	0.0014	49.53
ARMA(1, 1)-15	0.1	0.9	0.0288	0.0014	0.0040	0.1868	0.0046	0.0010	0.5057	0.0023	60.72
ARMA(1, 1)-16	0.5	−0.9	0.9999	0.9889	0.6514	0.0000	1.0000	0.9998	0.0025	0.7600	100.00
ARMA(1, 1)-17	0.5	−0.5	0.9996	0.9814	0.5326	0.0001	0.9999	0.9998	0.0021	0.6890	100.00
ARMA(1, 1)-18	0.5	0.1	0.8506	0.8386	0.1390	0.0323	0.9155	0.9559	0.0110	0.1406	99.63
ARMA(1, 1)-19	0.5	0.5	0.2377	0.2793	0.0309	0.0947	0.1737	0.2631	0.0805	0.0064	68.67
ARMA(1, 1)-20	0.5	0.9	0.0825	0.0136	0.0119	0.1141	0.0175	0.0088	0.2334	0.0016	40.75
ARMA(1, 1)-21	0.9	−0.9	1.0000	1.0000	0.9766	0.0000	1.0000	1.0000	0.0004	0.9990	100.00
ARMA(1, 1)-22	0.9	−0.5	1.0000	1.0000	0.9456	0.0000	1.0000	1.0000	0.0006	0.9992	100.00
ARMA(1, 1)-23	0.9	0.1	1.0000	0.9998	0.5421	0.0126	1.0000	1.0000	0.0008	0.9798	100.00
ARMA(1, 1)-24	0.9	0.5	0.9550	0.9889	0.1427	0.0602	0.9819	0.9956	0.0079	0.5708	99.99
ARMA(1, 1)-25	0.9	0.9	0.2931	0.3557	0.0315	0.0870	0.2446	0.3598	0.0738	0.0120	76.16

both coefficients are large and positive. For example, AR(2)-25 and ARMA-21 are schemes that consistently violate this rule.

Rule 4: 14 points in a row alternating up and down

Rule 4 corresponds to a series that is mean reverting. This is characteristic of an AR(1) scheme with negative coefficient. Thus, it is no surprise that this test is most often violated by the AR(1) and ARMA(1, 1) schemes with largely negative autoregressive coefficients, by AR(2) schemes with largely negative ϕ_1 and positive ϕ_2 and hardly ever by pure moving average schemes. Models AR(1)-1, AR(2)-5, and ARMA-3 are examples where this rule is effective.

Rule 5: 2 out of 3 points in a row in zone A or beyond on the same side of the center line

Rule 5 is an indicator of possible dependency. This rule is violated when a couple of points close together are very large, either positively or negatively. It is typically violated by AR(1) schemes when ϕ is large and in AR(2) schemes when $|\phi_1|$ and ϕ_2 are large. For example, AR(1)-6, AR(2)-25, MA(1)-1, MA(2)-2 and ARMA-21 are schemes causing this rule to be violated.

Rule 6: 4 out of 5 points in a row in zone B or beyond on the same side of the center line

Rule 6 is similar to Rule 5 in that it states that several points in a row were large, either positively or negatively. This also is indicative of dependency. This rule is typically violated by AR(1) schemes with a large coefficient and by AR(2) schemes when both coefficients are positive. It is also frequently violated by MA schemes with a largely negative θ_1 value as well as the combination of when ϕ_1 is large and θ_1 is negatively large for the ARMA processes. This rule is violated by models such as AR(1)-6, AR(2)-25, MA(1)-1, MA(2)-1 and ARMA-21.

Rule 7: 15 points in a row in zone C

Rule 7 corresponds to the observations falling too close to the center line for an extended period of time. This can be interpreted as an indication of dependency. This rule is typically violated when ϕ_1 is largely negative and infrequently when applied to series with moving average structure. For example, models AR(1)-1, AR(2)-5 and ARMA-3 cause this rule to be violated.

Rule 8: 8 points in a row not in zone C

Rule 8 can also be used to detect dependency in the data. It is typically violated when ϕ_2 is large for the AR(2) schemes and somewhat less frequently when θ is negative for the ARMA schemes. AR(1)-6, AR(2)-25 and ARMA-21 are examples of schemes that consistently violate this rule.

Overall, it appears that high levels of autocorrelation are effectively detected. Strong negative coefficient moving average structures also tend to violate the rules frequently. It is apparent, however, that series with weak to moderate dependencies, such as schemes AR(1)-3, AR(2)-7, MA(1)-4, MA(2)-10 and ARMA-4, tend to slip past the rules.

5. Recommendations

From the simulation results, it is evident that the sensitizing rules are not completely reliable for determining dependency. They do not pick up small degrees of autocorrelation and have a relatively high rate of falsely rejecting a series that is actually random. The original intent for these rules was to make it possible for a person on a factory floor to quickly determine if a process was out-of-control or not. However, with the current level of computer power, there exist more effective techniques for doing this job.

A simple way to show the correlation structure of a series is by its *Autocorrelation Function*.[7] From correlograms of observed series, we can see how strong the correlation is between time lags as well as how long it lasts. Such plots are useful in determining what, if any, autocorrelation is inherent in a realized series of observations. A plot where the autocorrelations do not come down to zero reasonably quickly indicates non-stationarity. The ACF of an MA(q) process "cuts off" at lag q, while the ACF of an AR(p) process attenuates slowly. An ARMA(p, q) process will also have an ACF plot that tends to decay out slowly.[4]

Figure 2 shows some autocorrelated series and their corresponding Sample Autocorrelation and Partial Autocorrelation Function plots as described in Sec. 2. The correlograms effectively show when a series' observations are not independent with significantly large spikes at some lags, as opposed to the NID case where there should be no significant spikes or patterns in the autocorrelations.

6. Conclusion

Each of the rules applied has its place in detecting for structure in a time series. No one rule is adequate in determining if the series is random or not. For instance, Rule 1, the easiest to apply, is only effective for certain types of autocorrelation. The rules that are effective simply look for characteristics of AR or MA schemes. Hence, how well a rule does is dependent on how strong the characteristic is. For example, the pattern searched for by Rule 4 is found in AR(1) models with a negative coefficient. The larger the negativity, the greater the proportion of violations found.

In conclusion, the sensitizing rules are not as effective in identifying moving average processes as they are for autoregressive series. This is not completely surprising as moving average processes are only correlated for a finite number q lags.

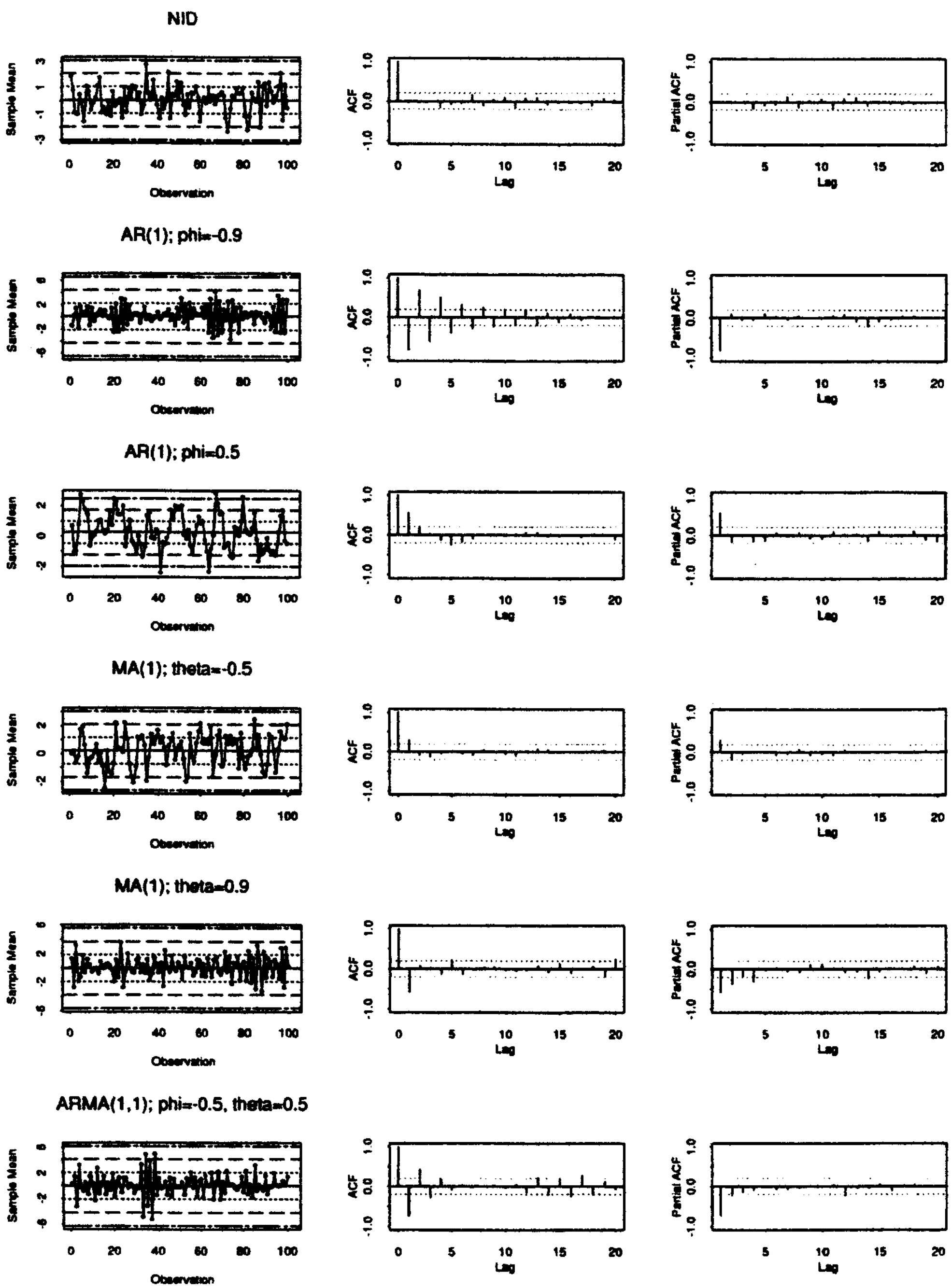

Fig. 2. Control chart with corresponding SACF and SPACF plots.

Most of the runs tests rely on a multi-point pattern as a means of violation detection. Through further experimentation, we also found that there is a high level of falsely classifying a series as out of control when using the sensitizing rules on long series. A possible alternative to the Shewhart Chart and sensitizing rules are SACF and SPACF plots which identify significant correlation between lagged points of the series. These plots are easy to obtain using almost any statistical package and should be considered for use in practice.

Acknowledgments

Dennis Lin is partially supported by the National Science Foundation via Grant DMS-9704711 and National Science Council of ROC via Contract NSC 87-2119-M-001-007.

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